Modern View of Light

- Photon = elementary particle

mass = 0

speed $c = 3 \times 10^{8}$ m/s

- Wave-particle duality

Energy $E = h\nu$

Momentum $p = h/\lambda$

h = Planck's constant

$= 6.626 \times 10^{-34}$ J s

$$\boxed{c = \lambda\nu}$$

Dispersion relation

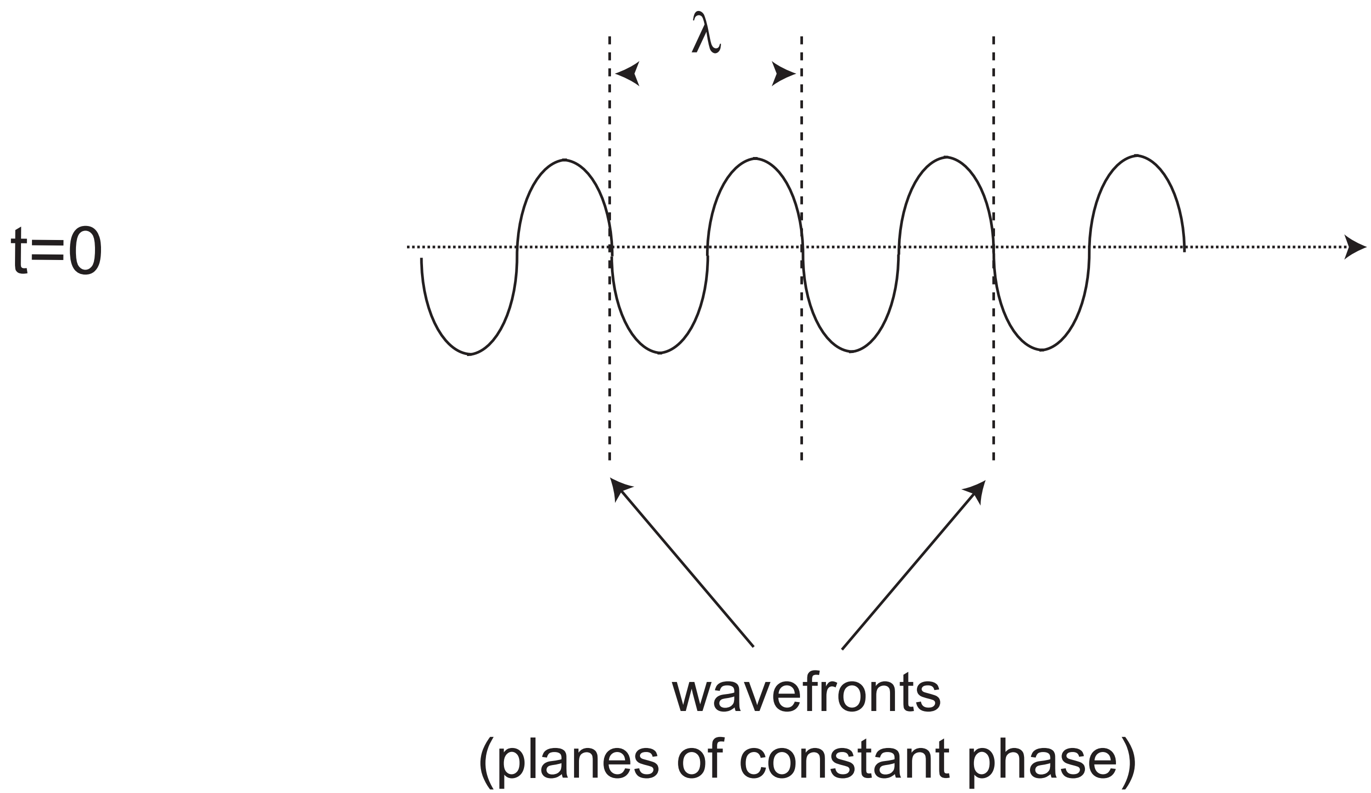
λ = wavelength (m)

spatial period of the light waves

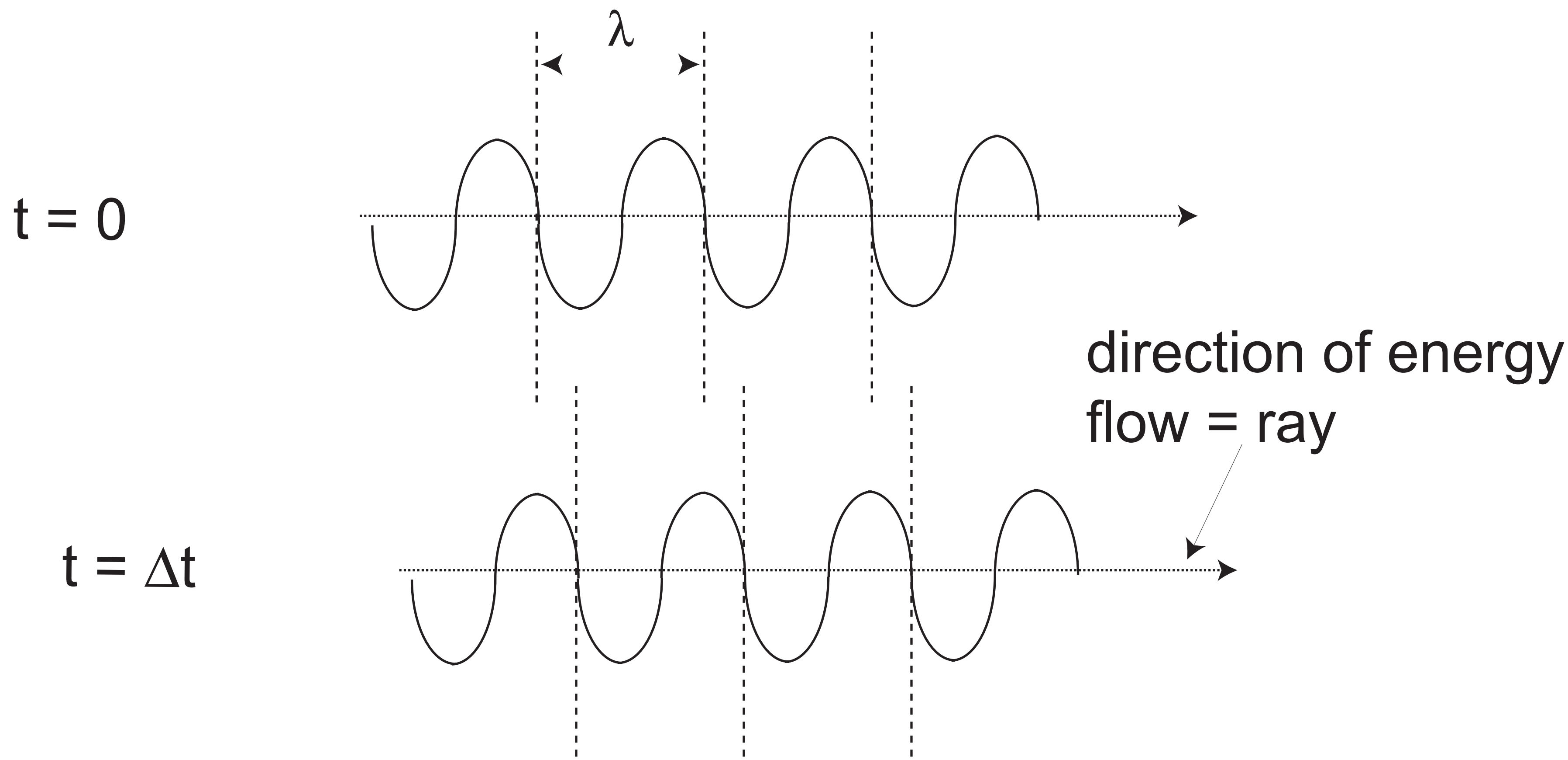
ν = frequency (1/s)

Temporal oscillation frequency of light waves

The concept of a "ray"

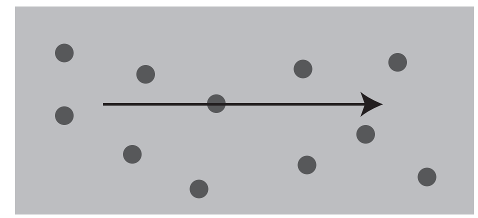


The concept of a "ray"



In homogeneous media,
light propagates in rectilinear paths

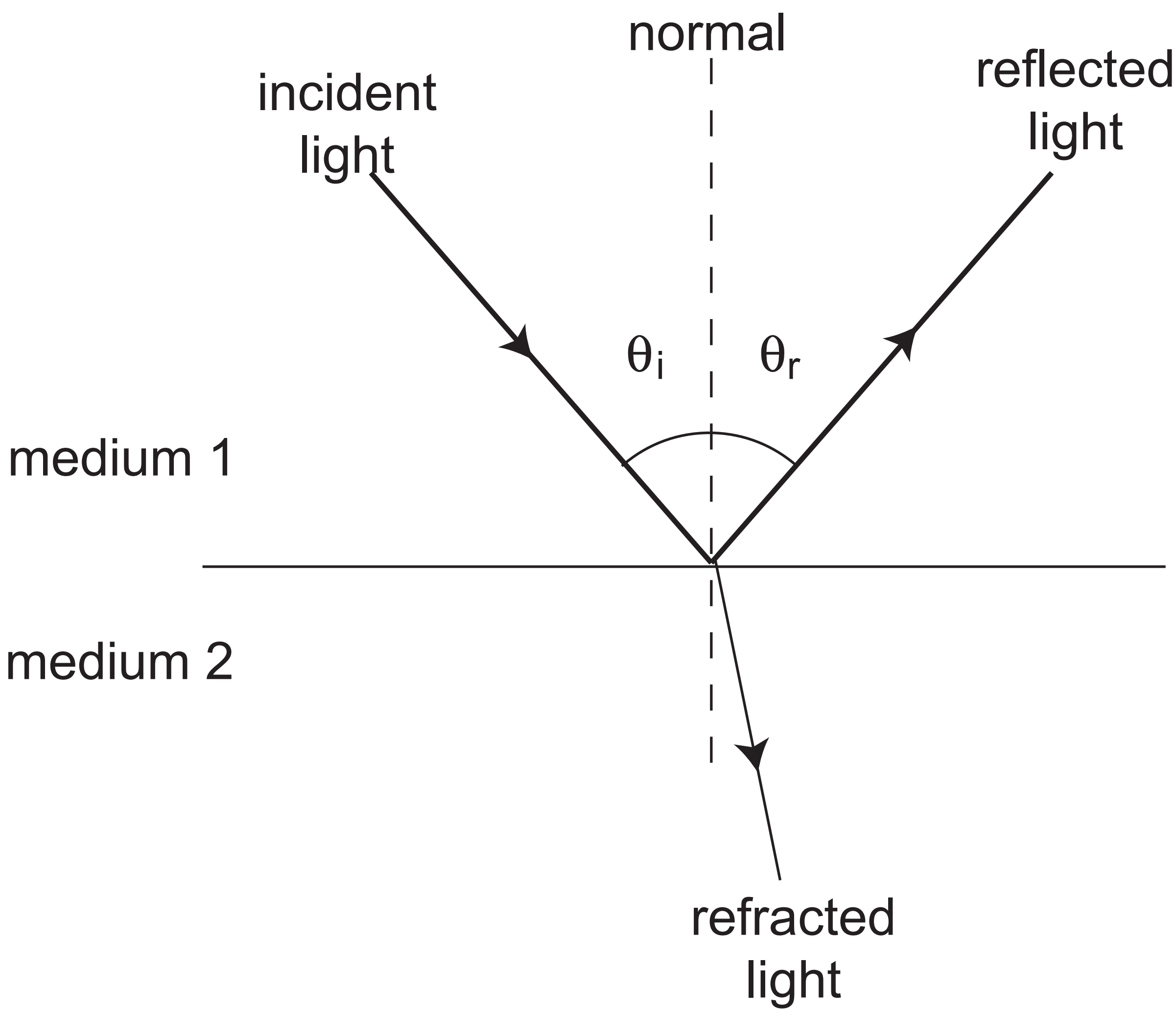
Light in matter



	vacuum	**matter**
Speed	$c = 3 \times 10^{8}$ m/s	$v = c / n$ n = index of refracton (or refractive index)
Absorption Coefficient	0	α energy transmitted through length $L = \exp(-2\alpha L)$

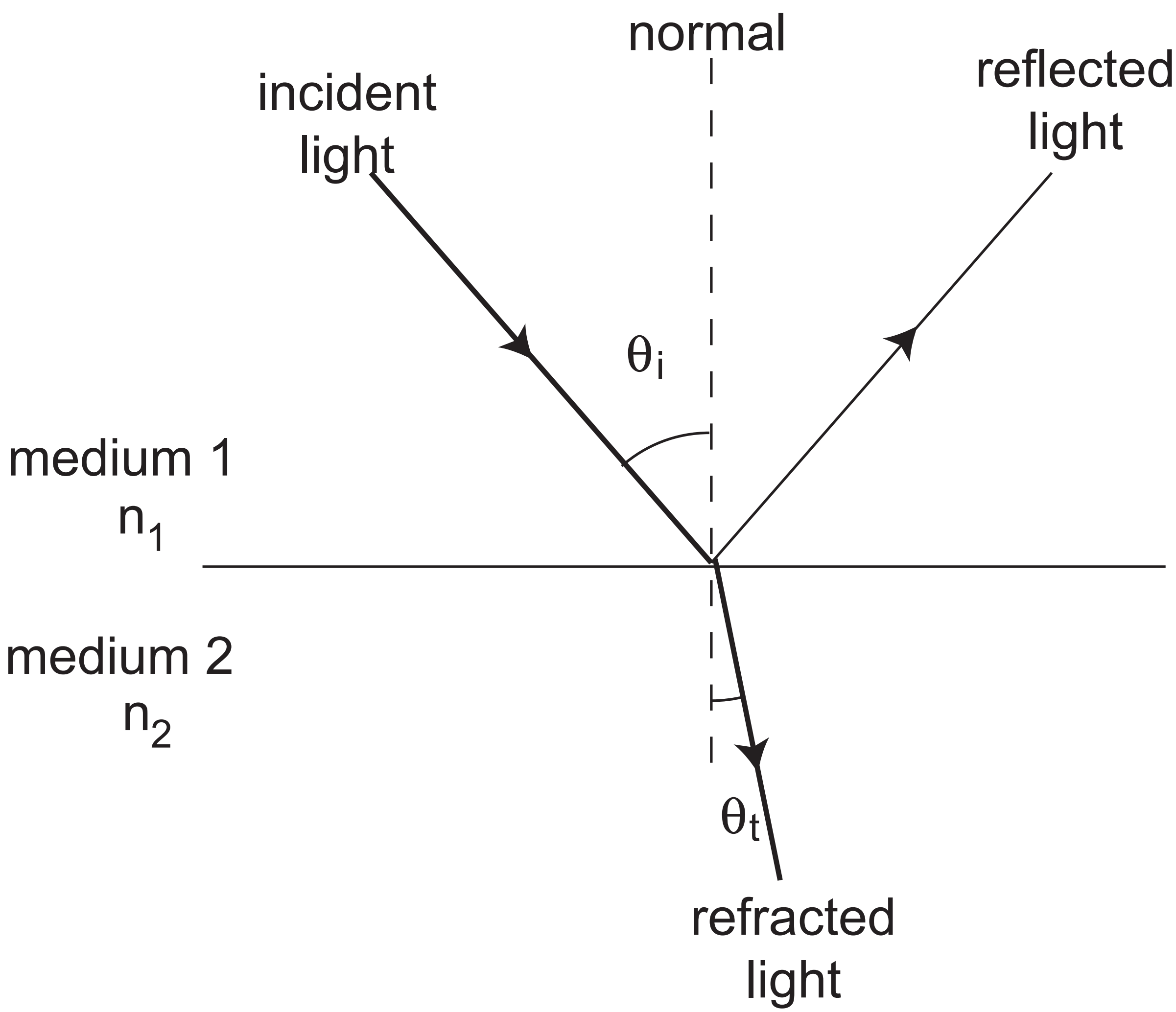
Example: Glass has $n \sim 1.5$, glass fiber has $\alpha \sim 0.0288$ /km

Reflection



- $\theta_i = \theta_r$
- Normal, incident & reflected rays lie in one plane

Refraction

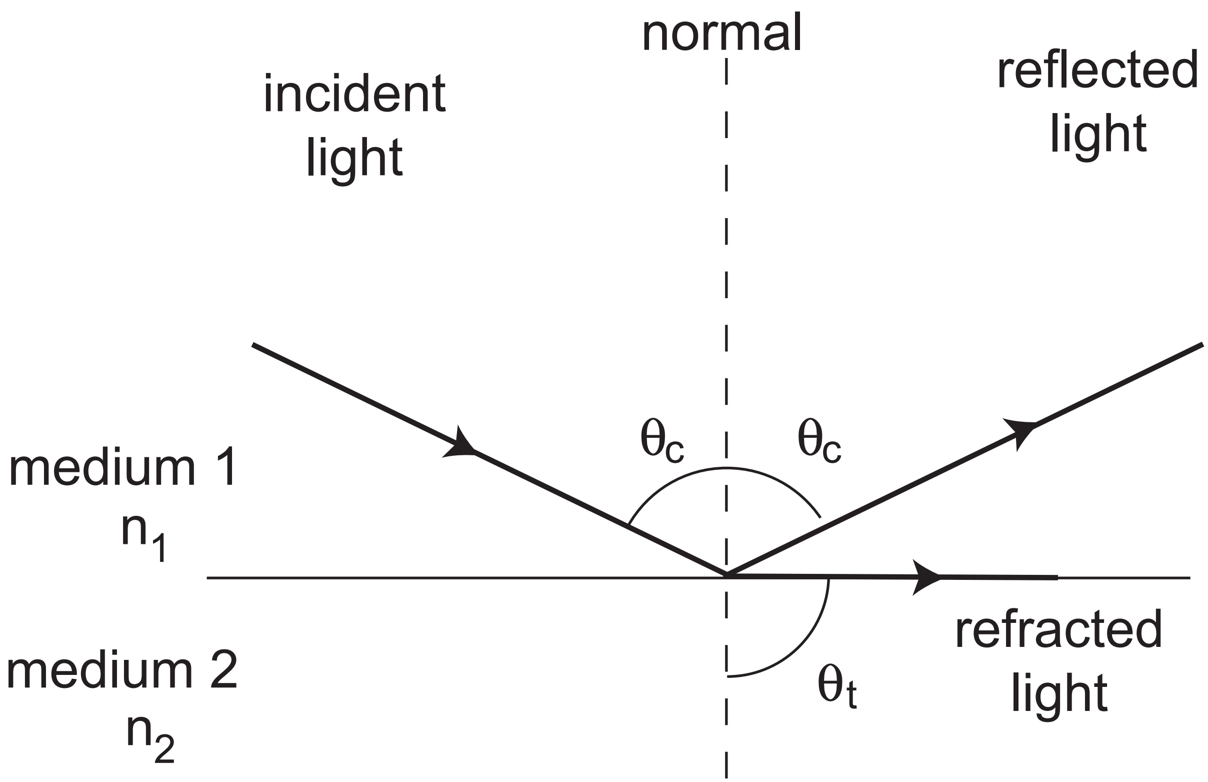


- $n_1 \sin\theta_i = n_2 \sin\theta_t$

  Snell's Law

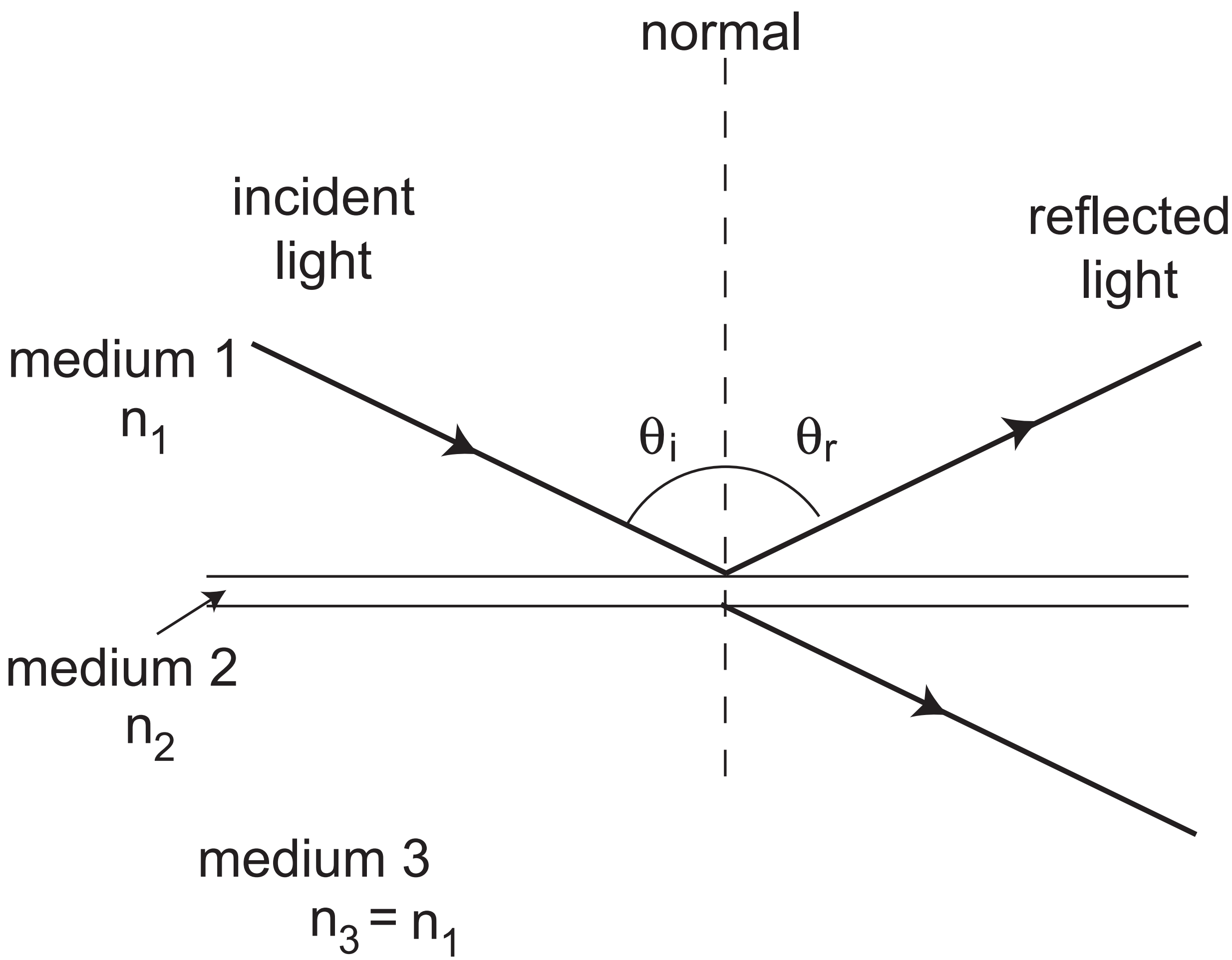
- Incident, reflected & refracted rays lie in one plane

Total Internal Reflection



- $n_1 > n_2$
- when $\theta_i = \theta_c = \sin^{-1} \dfrac{n_2}{n_1}$
  $\theta_t = 90^0$
- when $\theta_i > \theta_c$
  all light is reflected

Fustrated Total Internal Reflection (FTIR)



- $n_1 > n_2$
- $\theta_i > \theta_c$
  refracted beam is evanescent
- Light "tunnels" into 3rd medium

Geometrical Optics

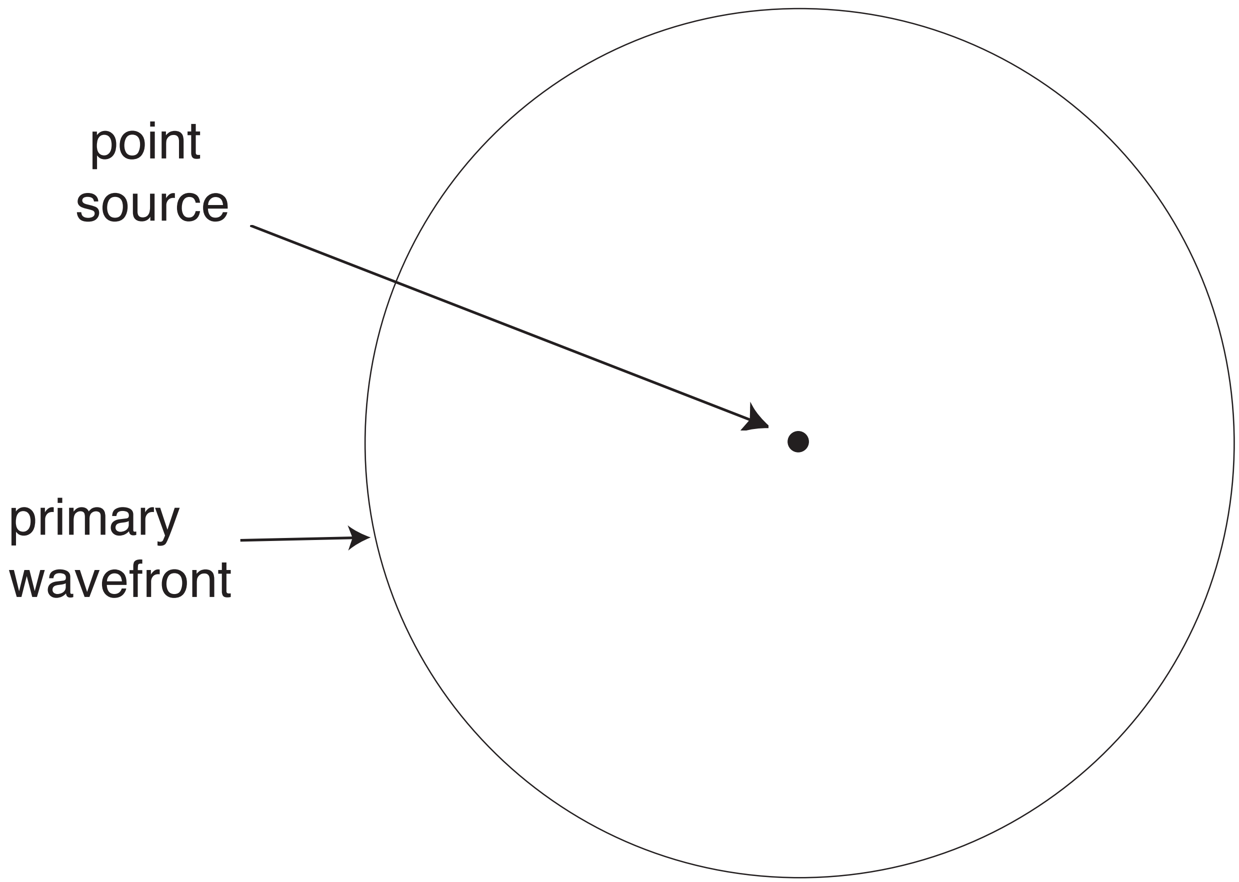
- λ, wavelength is small
- Wave effects such as interference & diffraction ignored
- Simple analysis, yet sufficient for many situations

. . . as opposed to

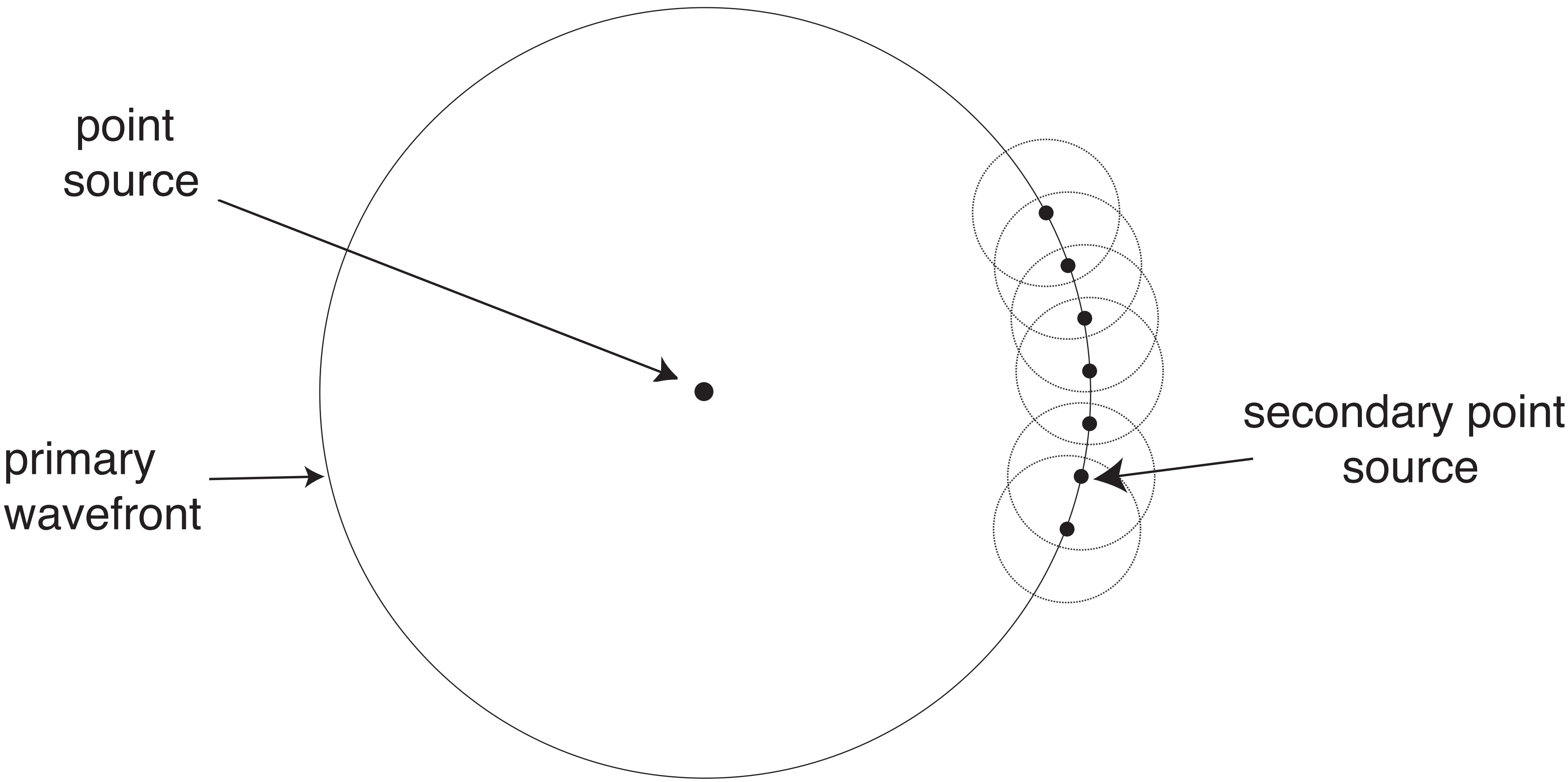
Physical Optics

- λ is non-zero

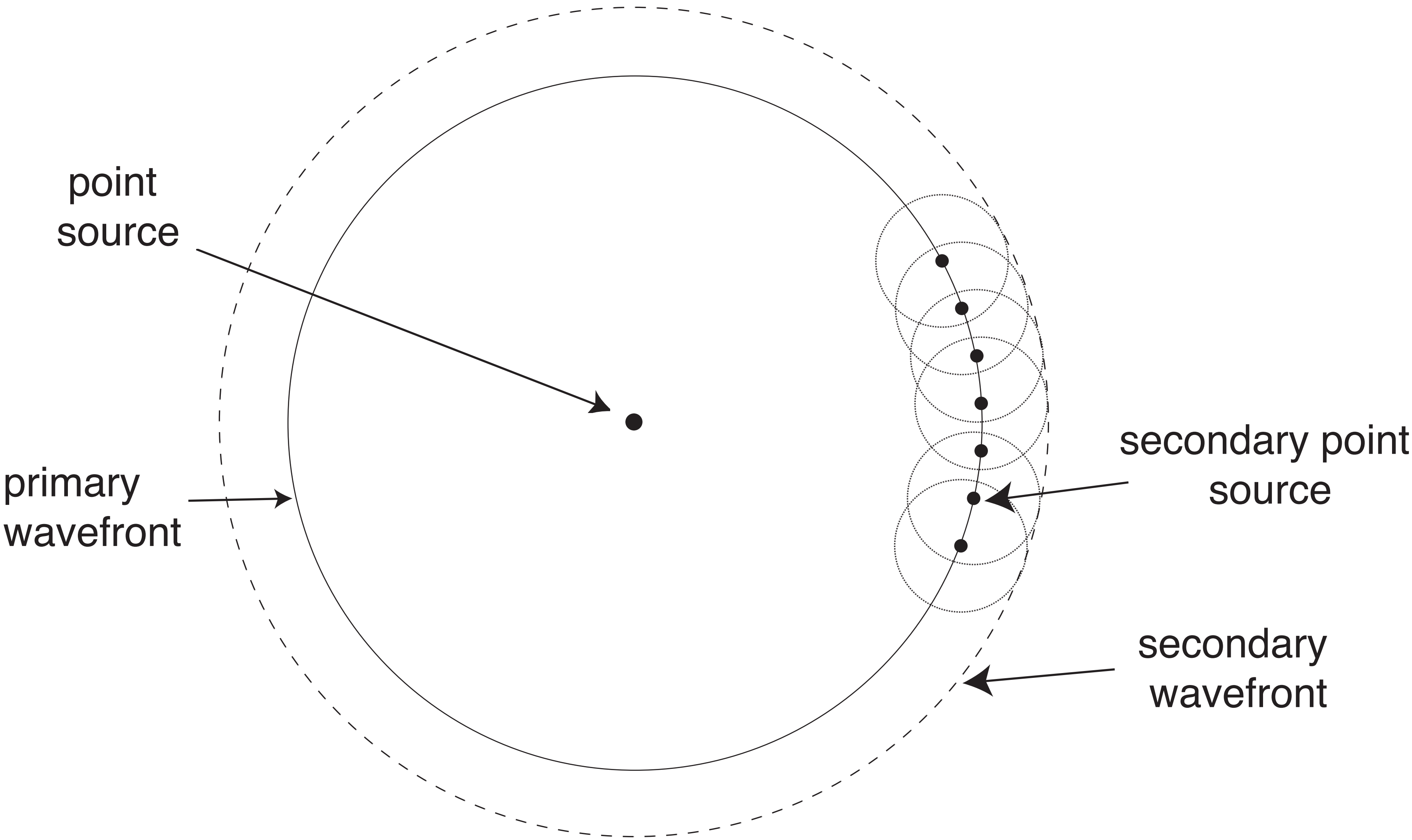
Huygen's Principle



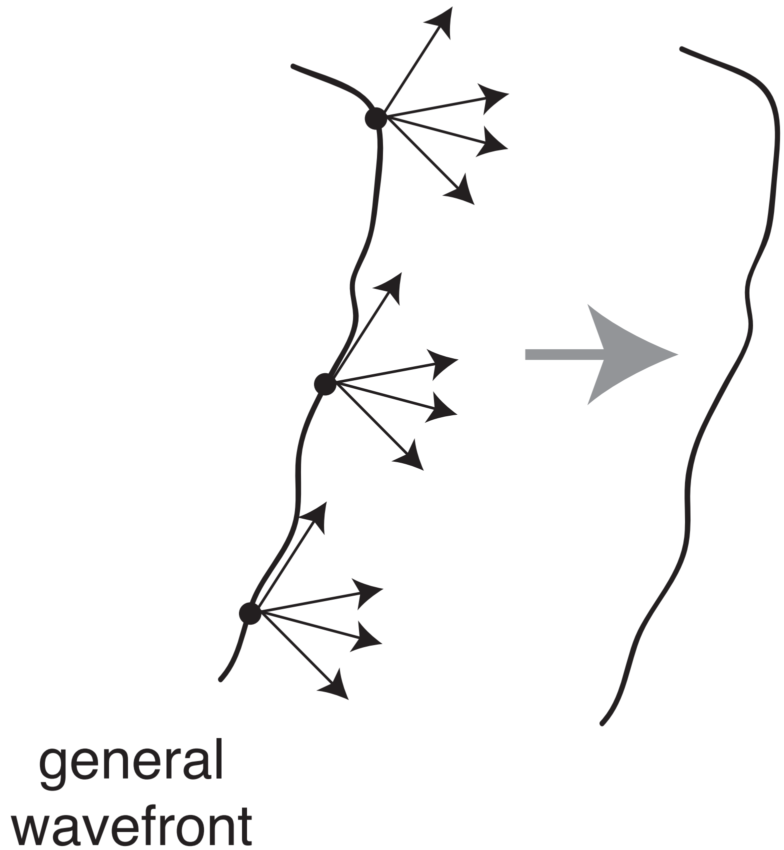
Huygen's Principle



Huygen's Principle



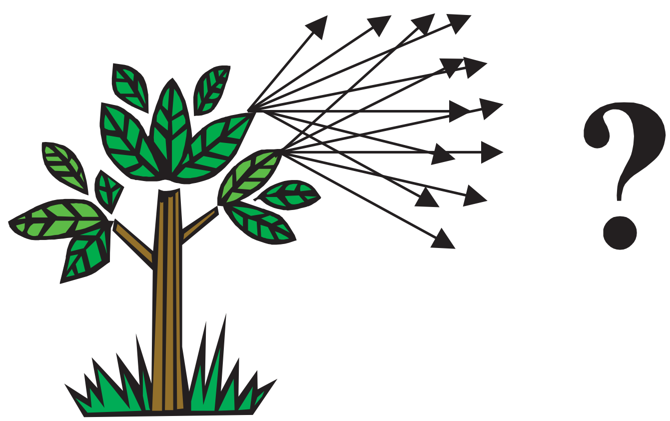
Huygen's Principle



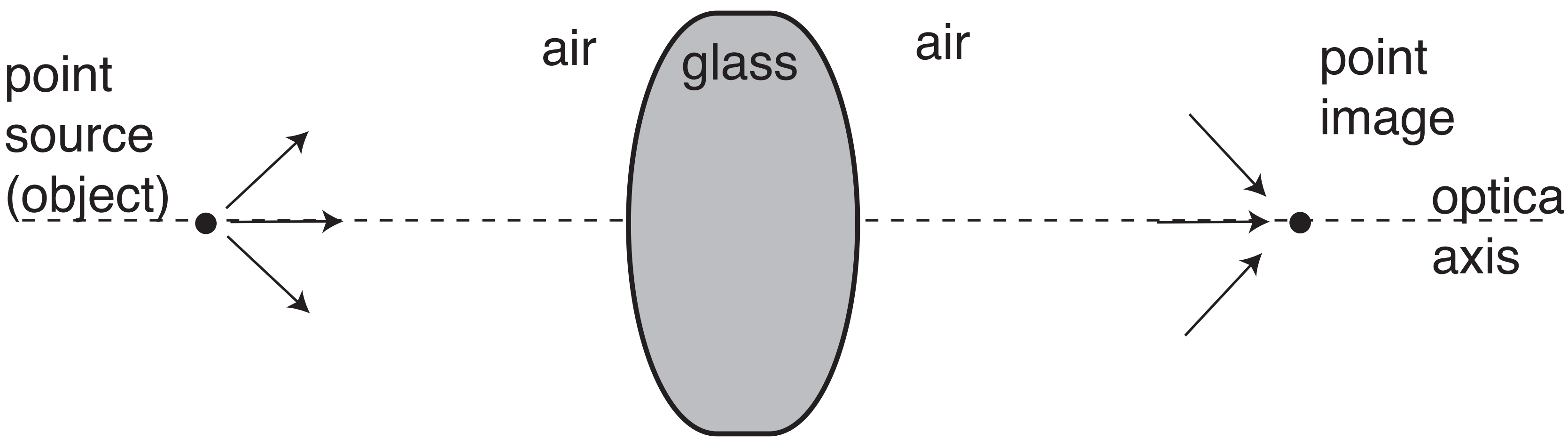
- Each point on a wavefront acts as a secondary point source emanating spherical wavelets.
- The wavefront after a short propagation distance is the superposition of all the spherical wavelets.

Why Imaging Systems are Necessary?

- Each point on an object scatters incident illumination into a spherical wavelet according to Huygen's Principle.
- At a short distance from the obect, the wavelets from all the points get entangled and object details are delocalized.
- The objective of imaging is to relocalize the object details by assigning ("focusing") rays from a single object point to a single "image" point.

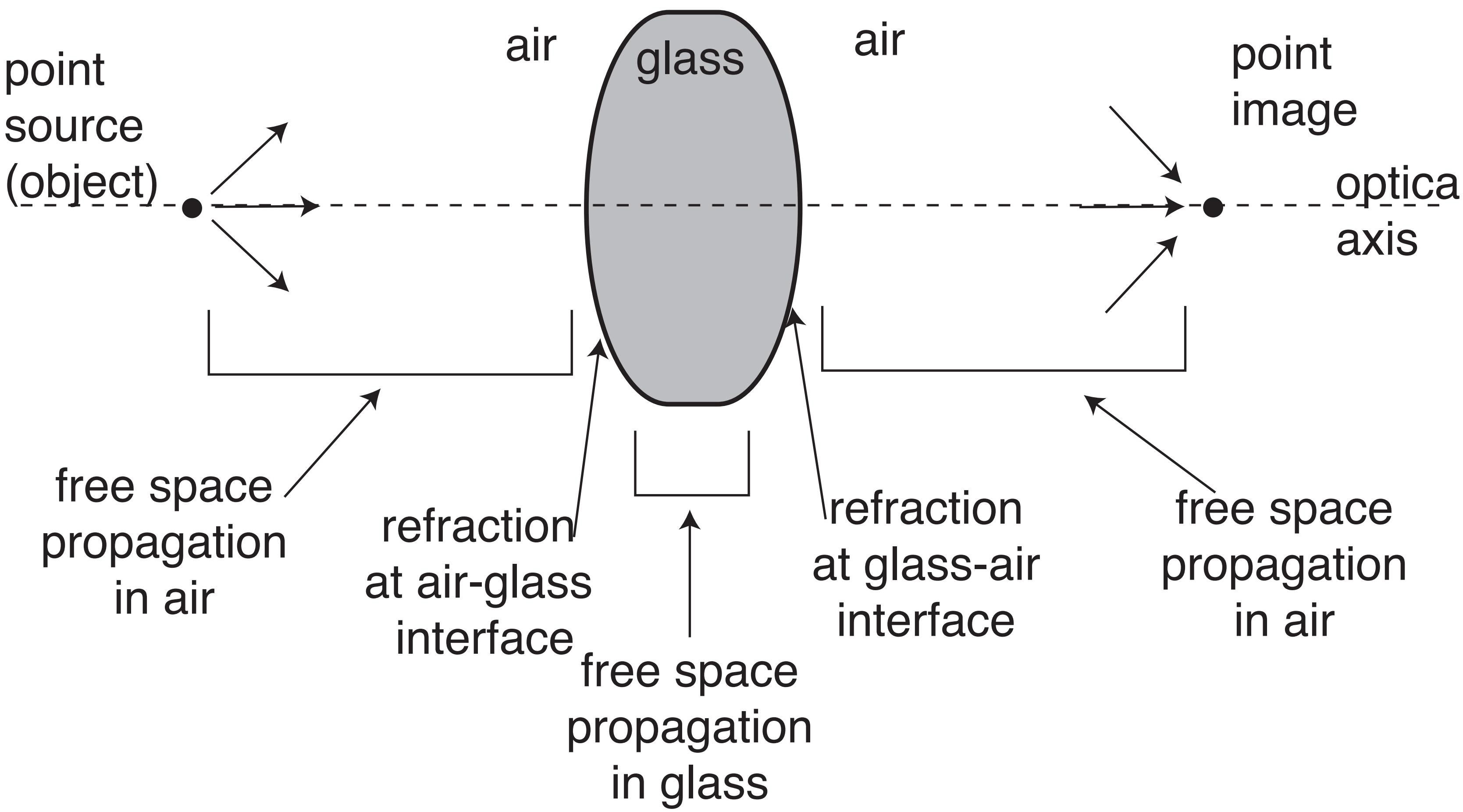


Lens: Main Instrument for Image Formation

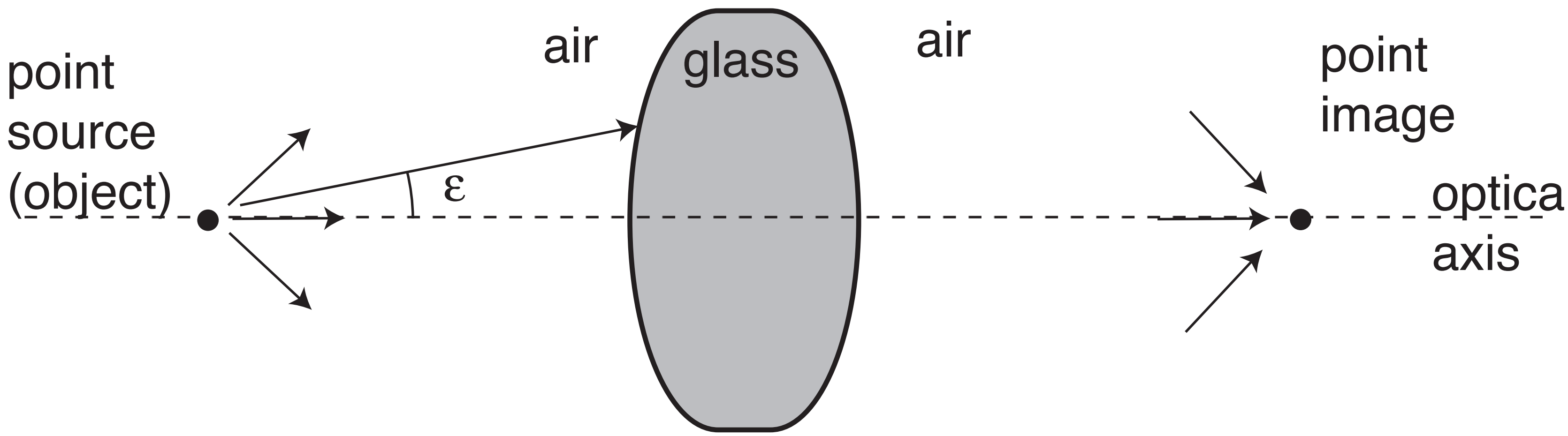


- The curved surface makes the rays bend at angles proportional to their distance from the optical axis, according to Snell's law. Thus a diverging wavefront becomes converging on the output side.

Analyzing Lenses: Ray Tracing



Paraxial Approximation



- Only rays close to the optical axis are considered

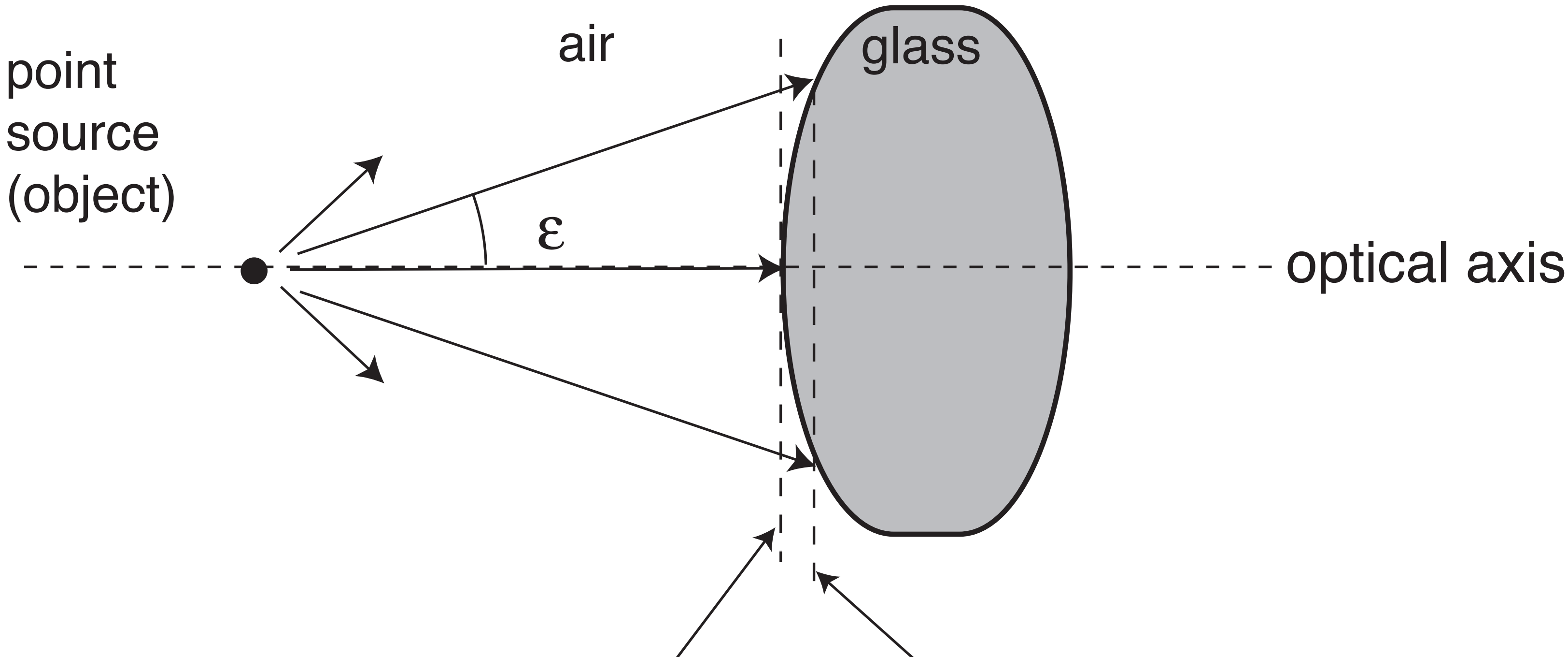
$$\boxed{\varepsilon << 1 \text{ rad}}$$

- 1st order Taylor approximations apply

$$\sin\varepsilon \approx \varepsilon \qquad \tan\varepsilon \approx \varepsilon \qquad \cos\varepsilon \approx 1$$

- Valid for ε upto 10-30 degrees

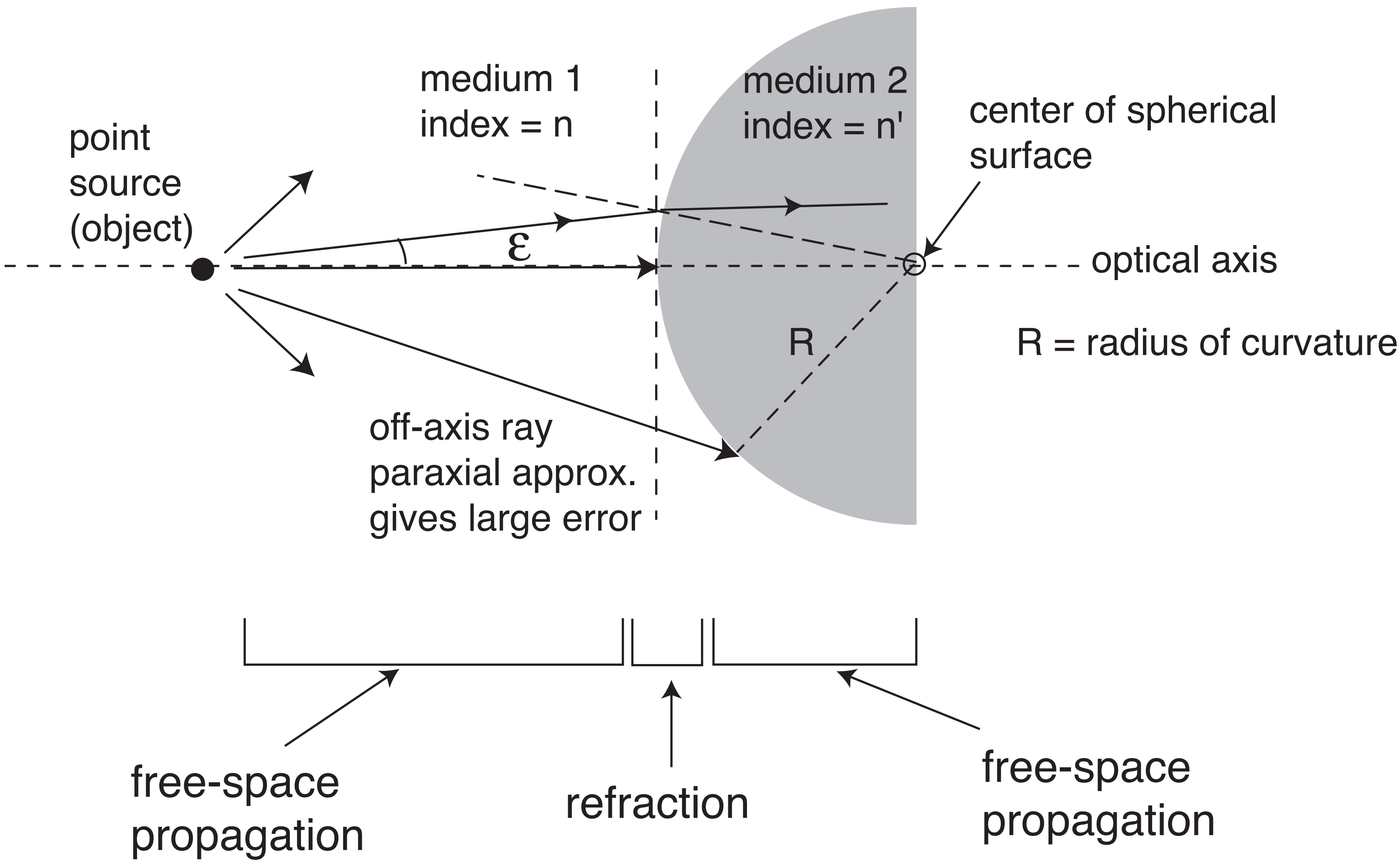
Paraxial Approximation



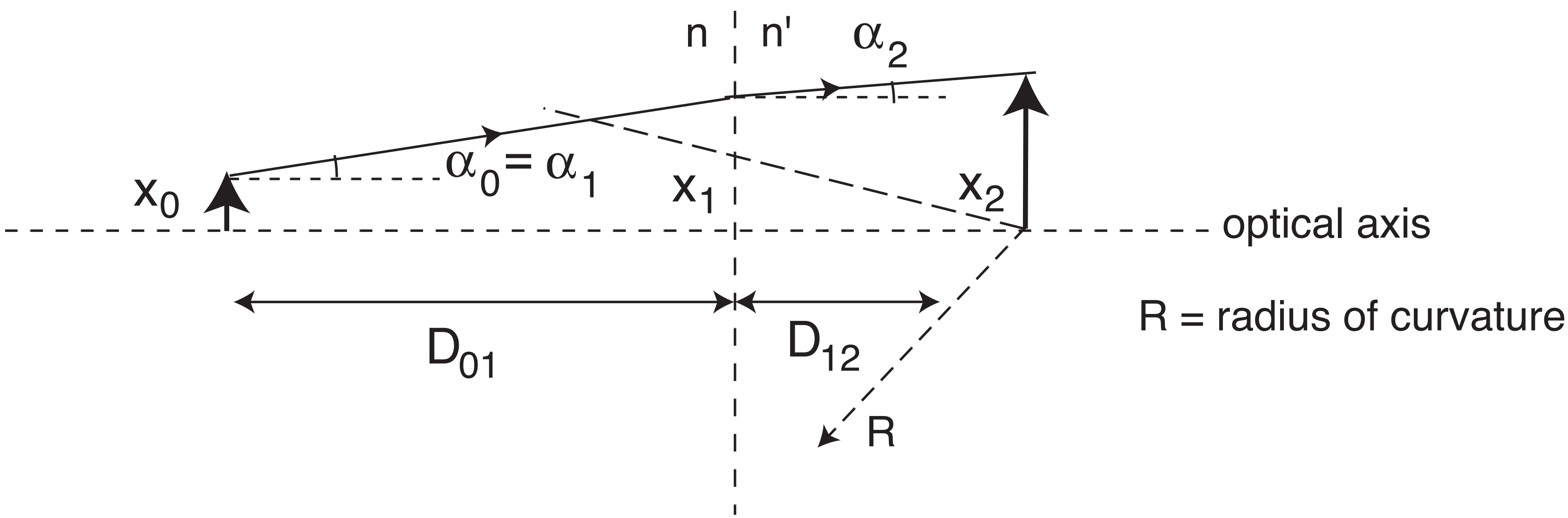
- Apply Snell's law assuming refraction occured at the intersection of the optical axis and the lens
- Ignore the distance between the actual off-axis ray intersection & the optical-axis intersection with the lens

Valid for small curvatures & thin optical elements

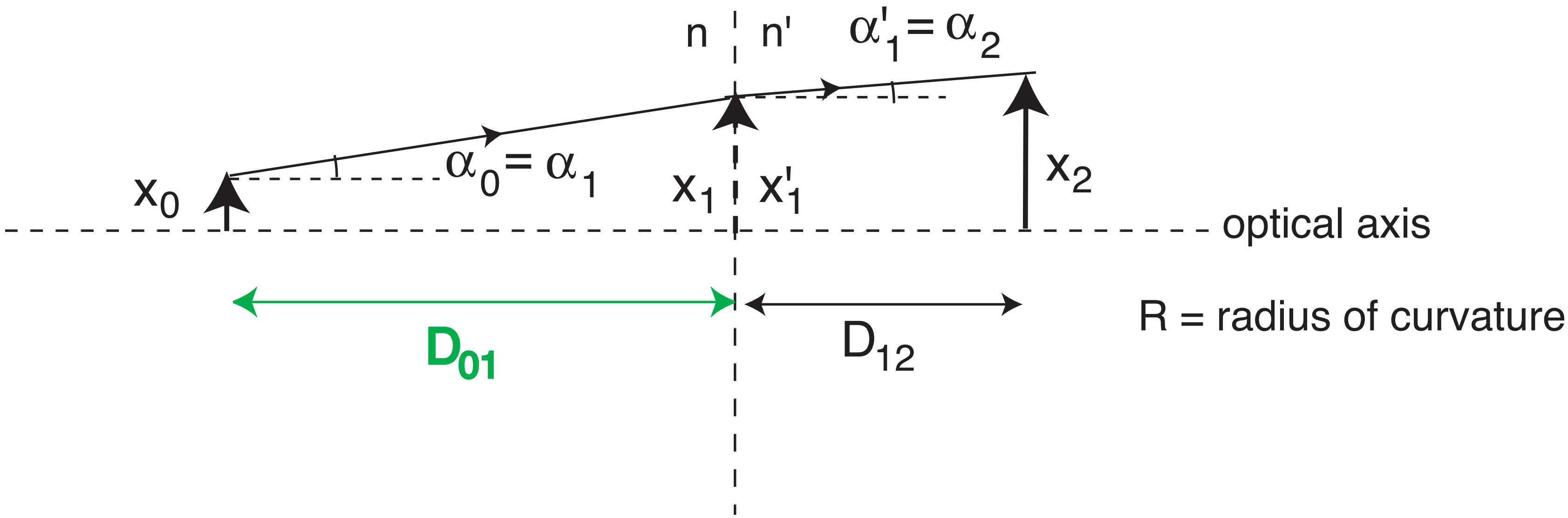
Example: 1 Sperical Surface



Example: 1 Sperical Surface



Example: 1 Sperical Surface

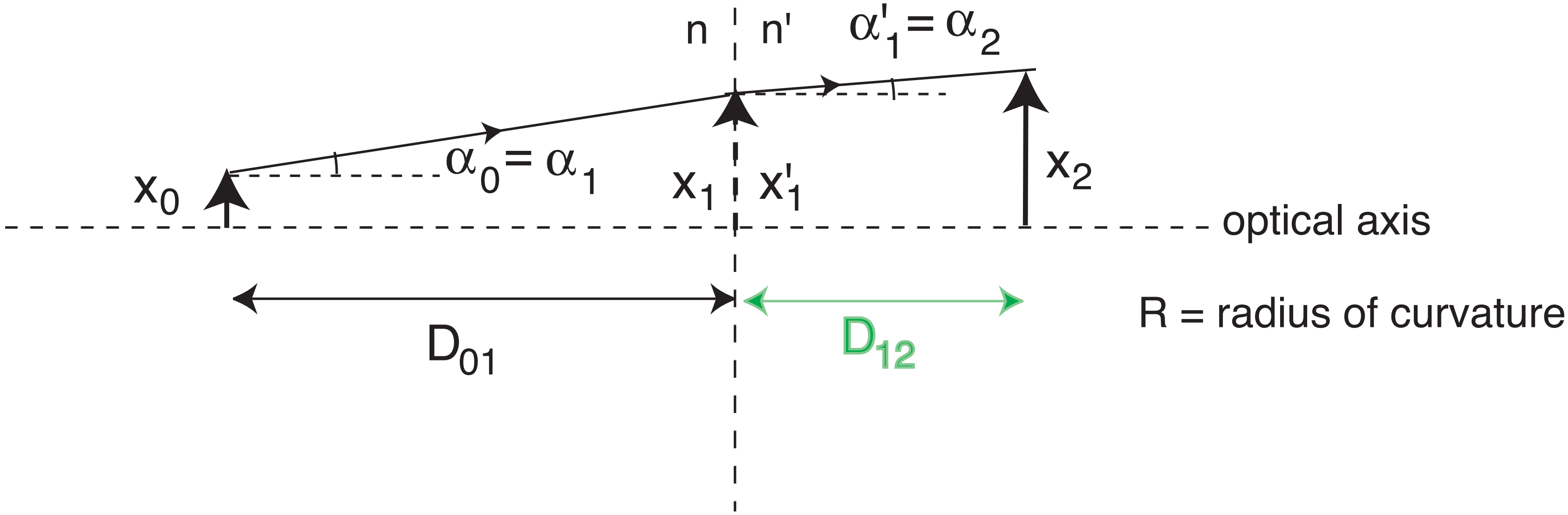


Starting Location: Position x_0 Direction α_0

Propagation through distance D_{01} $\left\{ \begin{array}{l} x_1 = x_0 + D_{01}\alpha_0 \\ \alpha_1 = \alpha_0 \end{array} \right.$

Refraction at spherical interface $\left\{ \begin{array}{l} x'_1 = x_1 \\ \alpha'_1 = \dfrac{n}{n'}\alpha_1 + \dfrac{n - n'}{n'R} x_1 \end{array} \right.$

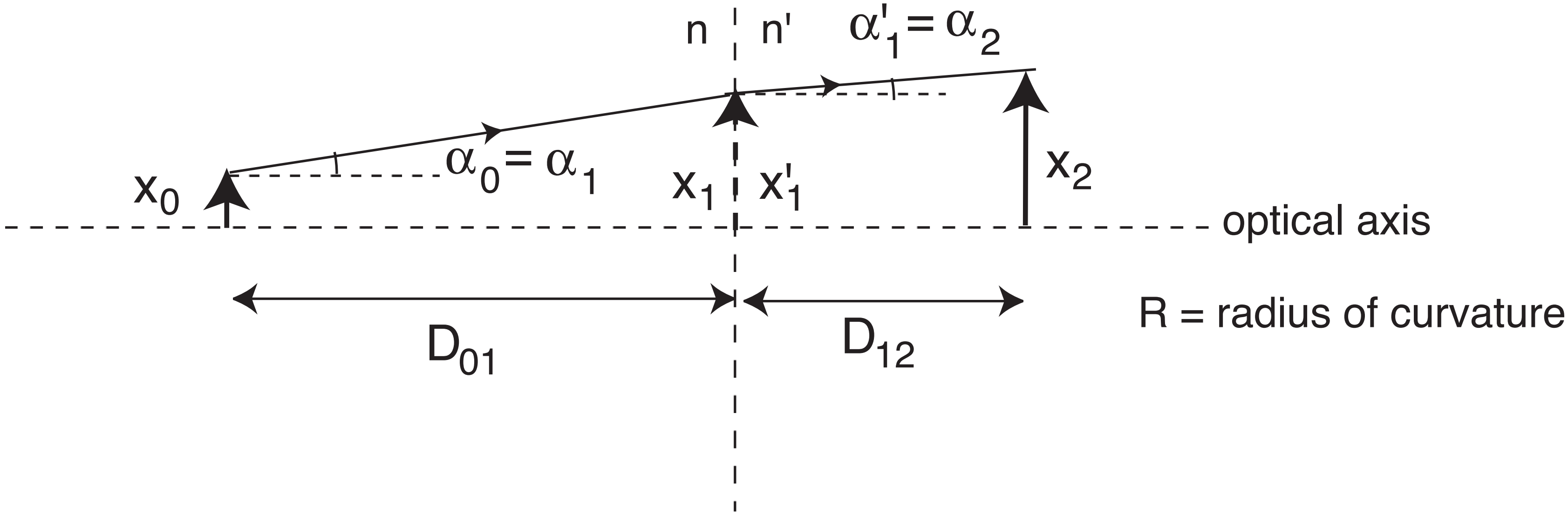
Example: 1 Sperical Surface



Propagation through distance D_{12}
$$\begin{cases} x_2 = x_1 + D_{12}\alpha'_1 \\ \alpha_2 = \alpha'_1 \end{cases}$$

Putting together . . .

Example: 1 Sperical Surface



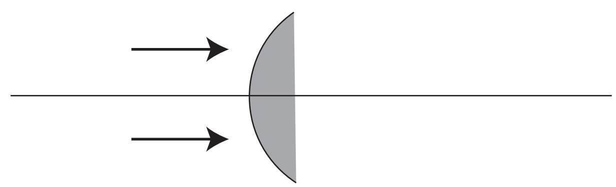
$$\mathbf{x_2} = \left(\frac{n - n'}{n'}\frac{D_{12}}{R} + 1\right)\mathbf{x_0} + \left(D_{01} + \frac{nD_{12}}{n'} + \frac{n - n'}{n'}\frac{D_{01}D_{12}}{R}\right)\boldsymbol{\alpha_0}$$

$$\boldsymbol{\alpha_2} = \left(\frac{n - n'}{n'R}\right)\mathbf{x_0} + \left(\frac{n}{n'} + \frac{n - n'}{n'}\frac{D_{10}}{R}\right)\boldsymbol{\alpha_0}$$

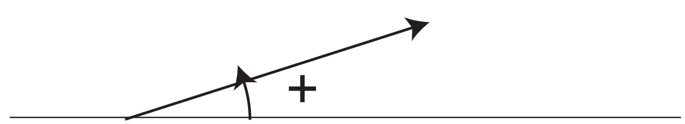
Sign Conventions

- Light travels from left to right

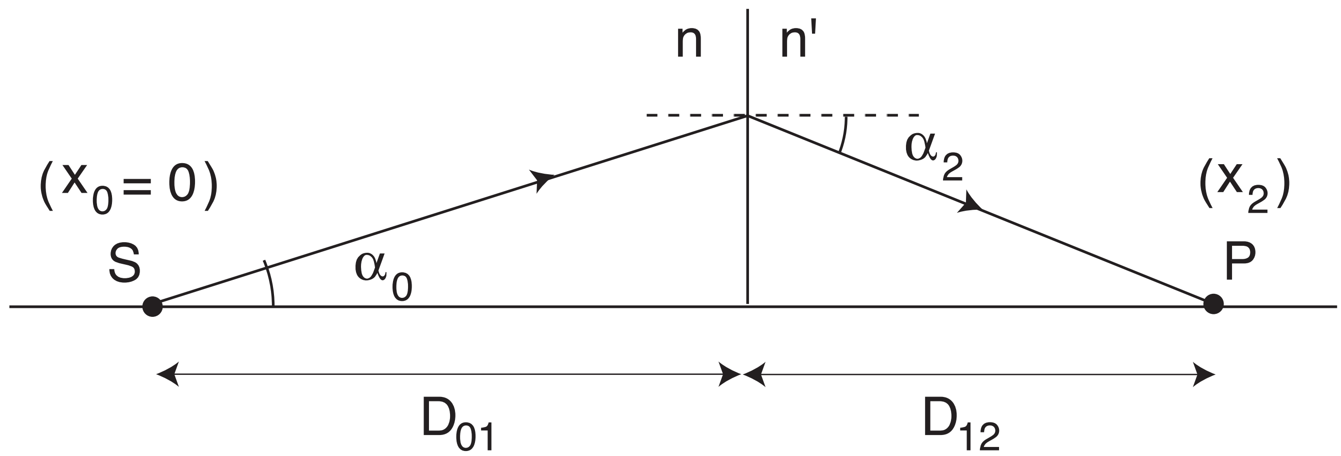
- Radius of curvature is positive when surface is convex towards left



- Longitudinal distances are positive if pointing to the right
- Lateral distances are positive when pointing up
- Ray angles are positive if ray direction is obtained by rotating the optical axis (+z) counter-clockwise through an acute angle



On-axis Image Formation



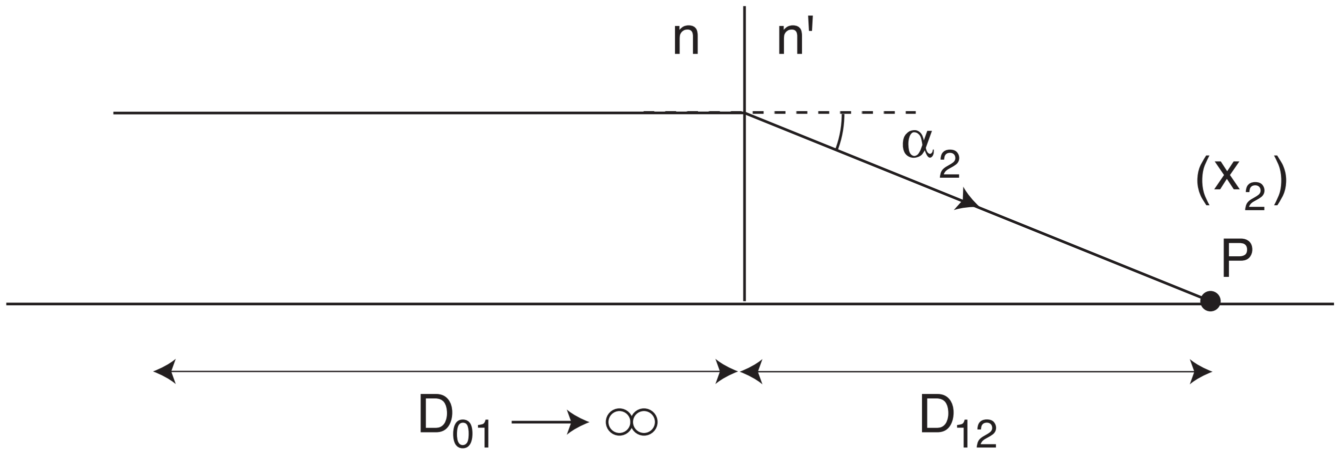
- All rays emanating from S converge to P irrespective of angle, α_0

$$\longrightarrow \quad \frac{\partial x_2}{\partial \alpha_0} = 0$$

$$\longrightarrow \quad \frac{n'}{D_{12}} + \frac{n}{D_{01}} = \frac{n' - n}{R}$$

"power" of spherical surface
(units= Diopters; 1D = 1/m)

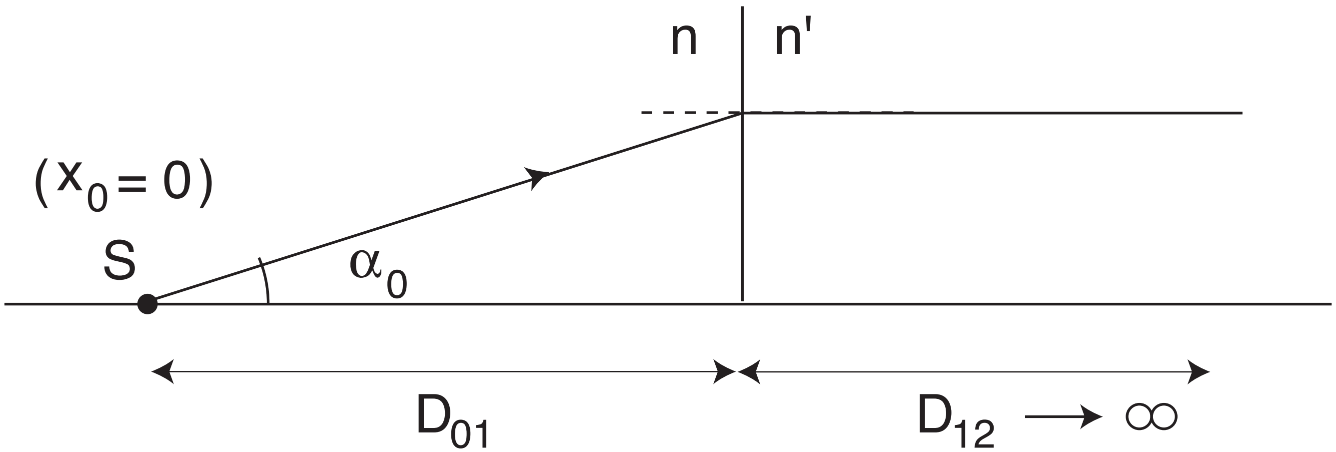
Image of Point Object at Infinity



$$D_{12} = f' = \frac{n' R}{n' - n}$$

Image Focal Length

Point Object Imaged at Infinity



$$D_{01} = f = \frac{n\,R}{n' - n}$$

Object Focal Length

Matrix Formulation

$$\begin{bmatrix} n_{out}\,\alpha_{out} \\ x_{out} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} n_{in}\,\alpha_{in} \\ x_{in} \end{bmatrix}$$

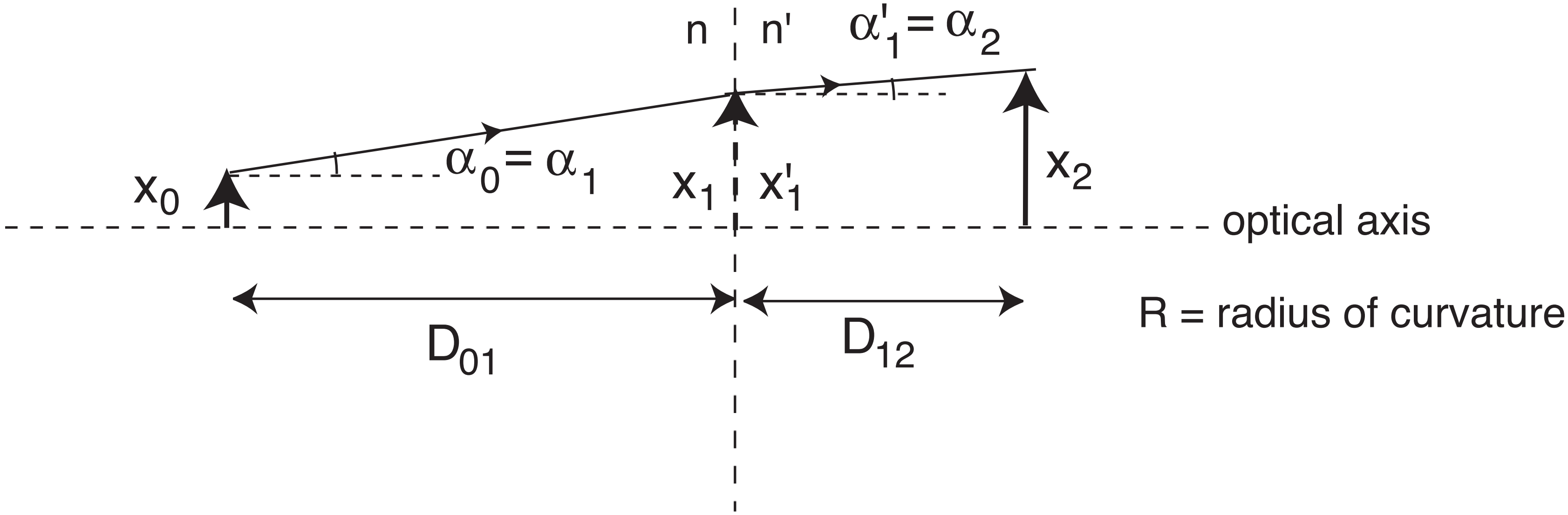
Translation through Uniform Medium

$$\begin{bmatrix} n\,\alpha_1 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{D_{01}}{n} & 1 \end{bmatrix} \begin{bmatrix} n\,\alpha_0 \\ x_0 \end{bmatrix}$$

Refraction by Spherical Surface

$$\begin{bmatrix} n'\,\alpha'_1 \\ x'_1 \end{bmatrix} = \begin{bmatrix} 1 & -\left(\overset{\text{power}}{\frac{n'-n}{R}}\right) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} n\,\alpha_1 \\ x_1 \end{bmatrix}$$

Example: 1 Sperical Surface

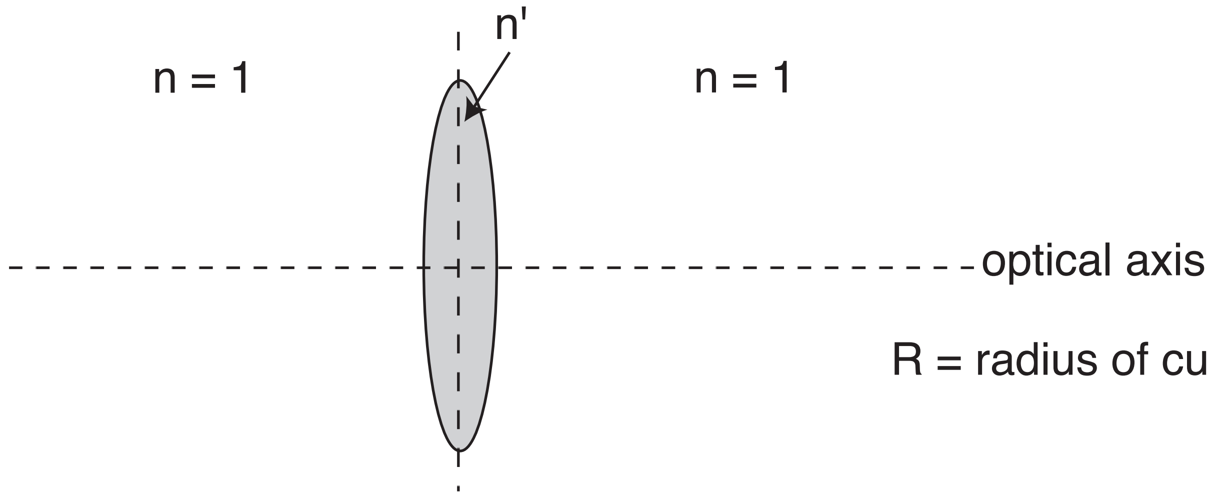


$$\begin{bmatrix} \mathbf{n'\alpha'_2} \\ \mathbf{x'_2} \end{bmatrix} = \begin{bmatrix} \text{Translation through } D_{01} \end{bmatrix} \times \begin{bmatrix} \text{Refraction at spherical interface} \end{bmatrix} \times \begin{bmatrix} \text{Translation through } D_{12} \end{bmatrix} \times \begin{bmatrix} \mathbf{n\,\alpha_0} \\ \mathbf{x_0} \end{bmatrix}$$

$$\mathbf{x_2} = \left(\frac{n - n'}{n'}\frac{D_{12}}{R} + 1\right)\mathbf{x_0} + \left(D_{01} + \frac{nD_{12}}{n'} + \frac{n - n'}{n'}\frac{D_{01}D_{12}}{R}\right)\boldsymbol{\alpha_0}$$

$$\boldsymbol{\alpha_2} = \left(\frac{n - n'}{n'R}\right)\mathbf{x_0} + \left(\frac{n}{n'} + \frac{n - n'}{n'}\frac{D_{10}}{R}\right)\boldsymbol{\alpha_0}$$

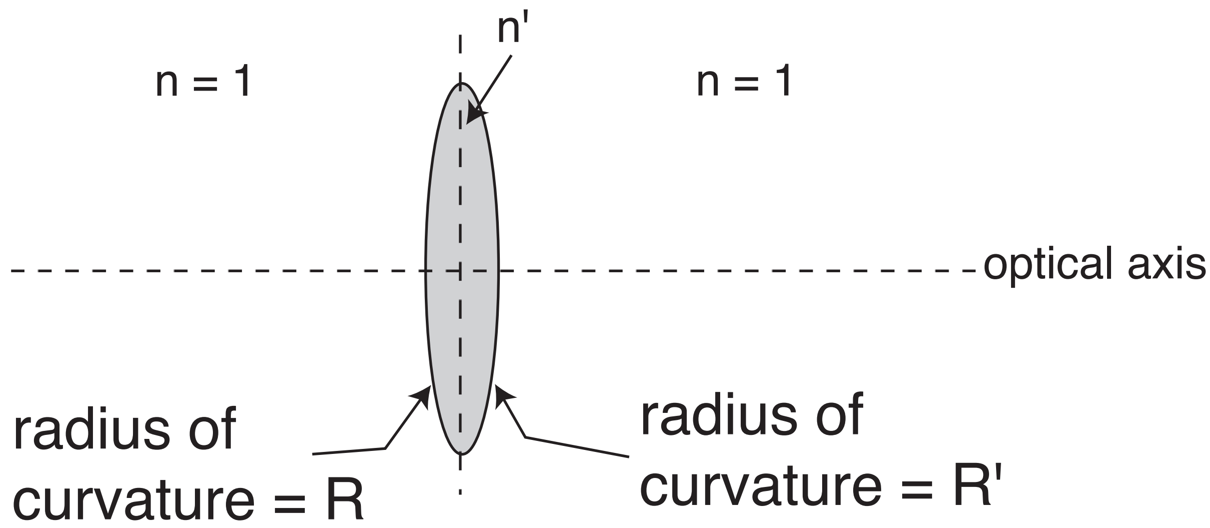
Thin Lens



R = radius of curvature

$$\begin{bmatrix}\alpha'_{out}\\ x'_{out}\end{bmatrix} = \begin{bmatrix}\text{Refraction at}\\ \text{1st spherical}\\ \text{interface}\end{bmatrix} \text{X} \begin{bmatrix}\text{Refraction at}\\ \text{2nd spherical}\\ \text{interface}\end{bmatrix} \text{X} \begin{bmatrix}\alpha_{in}\\ x_{in}\end{bmatrix}$$

Thin Lens



$$\begin{bmatrix} \alpha'_{out} \\ x'_{out} \end{bmatrix} = \begin{bmatrix} 1 & -\left(\frac{1 - n'}{R'}\right) \\ 0 & 1 \end{bmatrix} \mathrm{X} \begin{bmatrix} 1 & -\left(\frac{n' - 1}{R}\right) \\ 0 & 1 \end{bmatrix} \mathrm{X} \begin{bmatrix} \alpha_{in} \\ x_{in} \end{bmatrix}$$

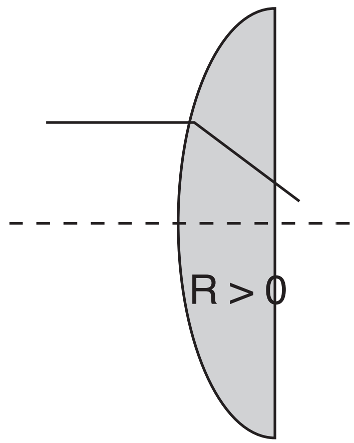
$$= \begin{bmatrix} 1 & -\left(\frac{n' - 1}{R} + \frac{1 - n'}{R'}\right) \\ 0 & 1 \end{bmatrix}$$

$$P_{thin\text{-}lens} = (n' - 1)\left(\frac{1}{R} - \frac{1}{R'}\right)$$

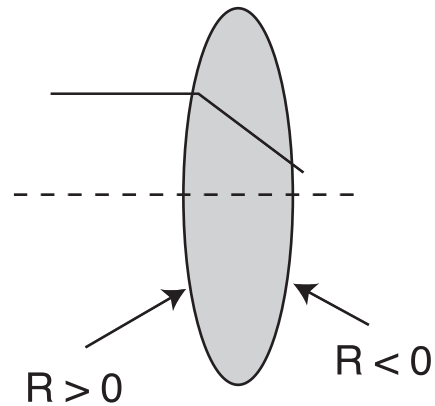
Lens-maker's formula

Power of Surfaces

- Positive power bends rays "inwards"

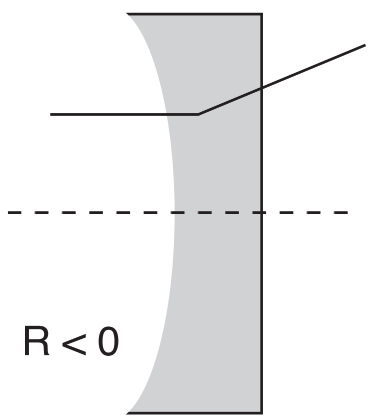


Plano-convex lens

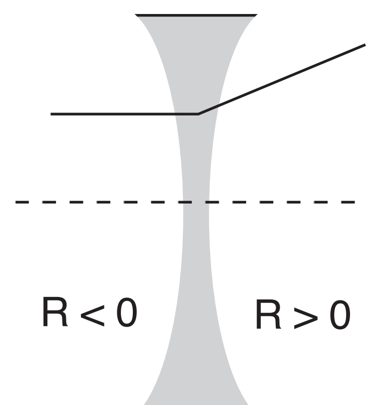


Bi-convex Lens

- Negative power bends rays "outwards"



Plano-concave lens



Bi-concave Lens

Power of Surfaces

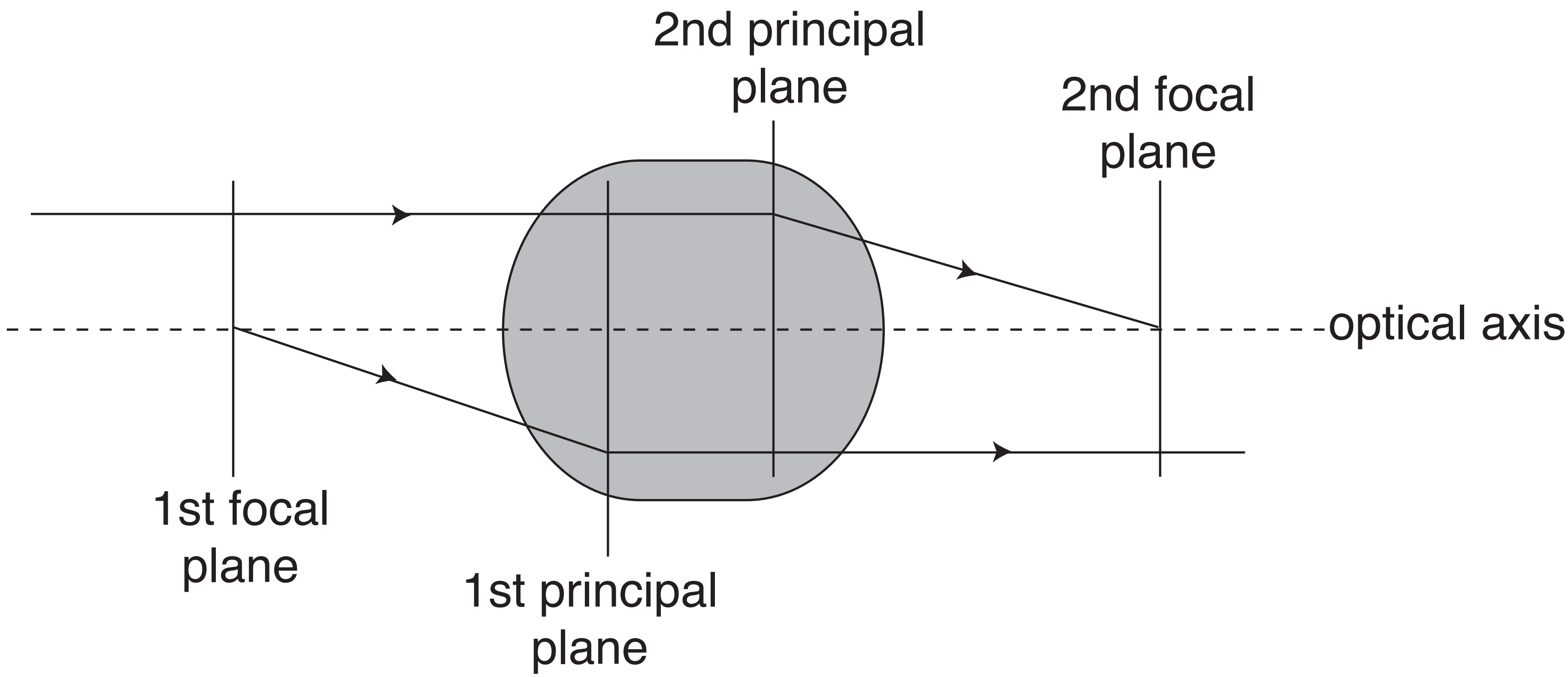
- Matrix Formulation

$$\text{Power} = -M_{12}$$

- Power & Focal Length

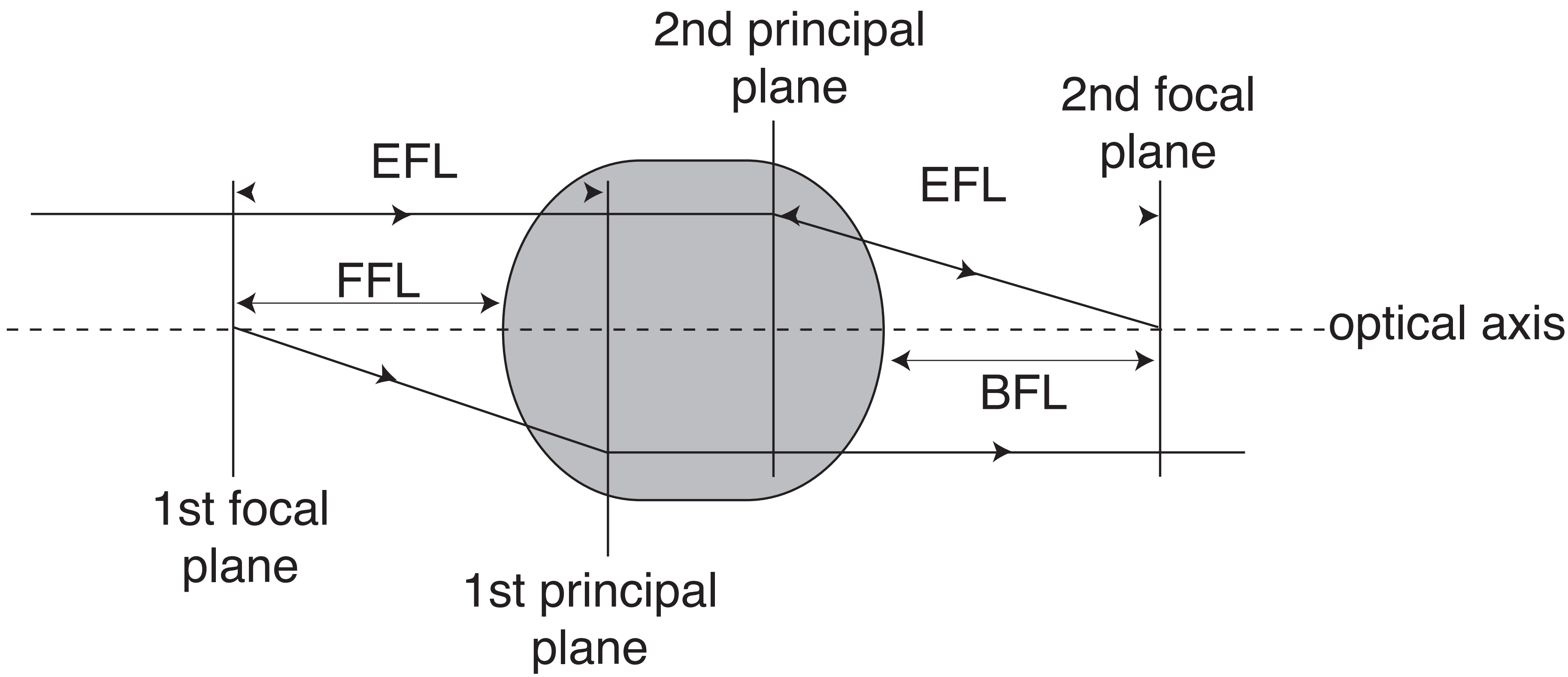
$$f = 1 / \text{power}$$

Thick Lens: Principal Planes



Note: In paraxial approximation, principal and focal planes are flat, whereas in reality these are curved surfaces (not spherical).

Thick Lens: Focal Lengths

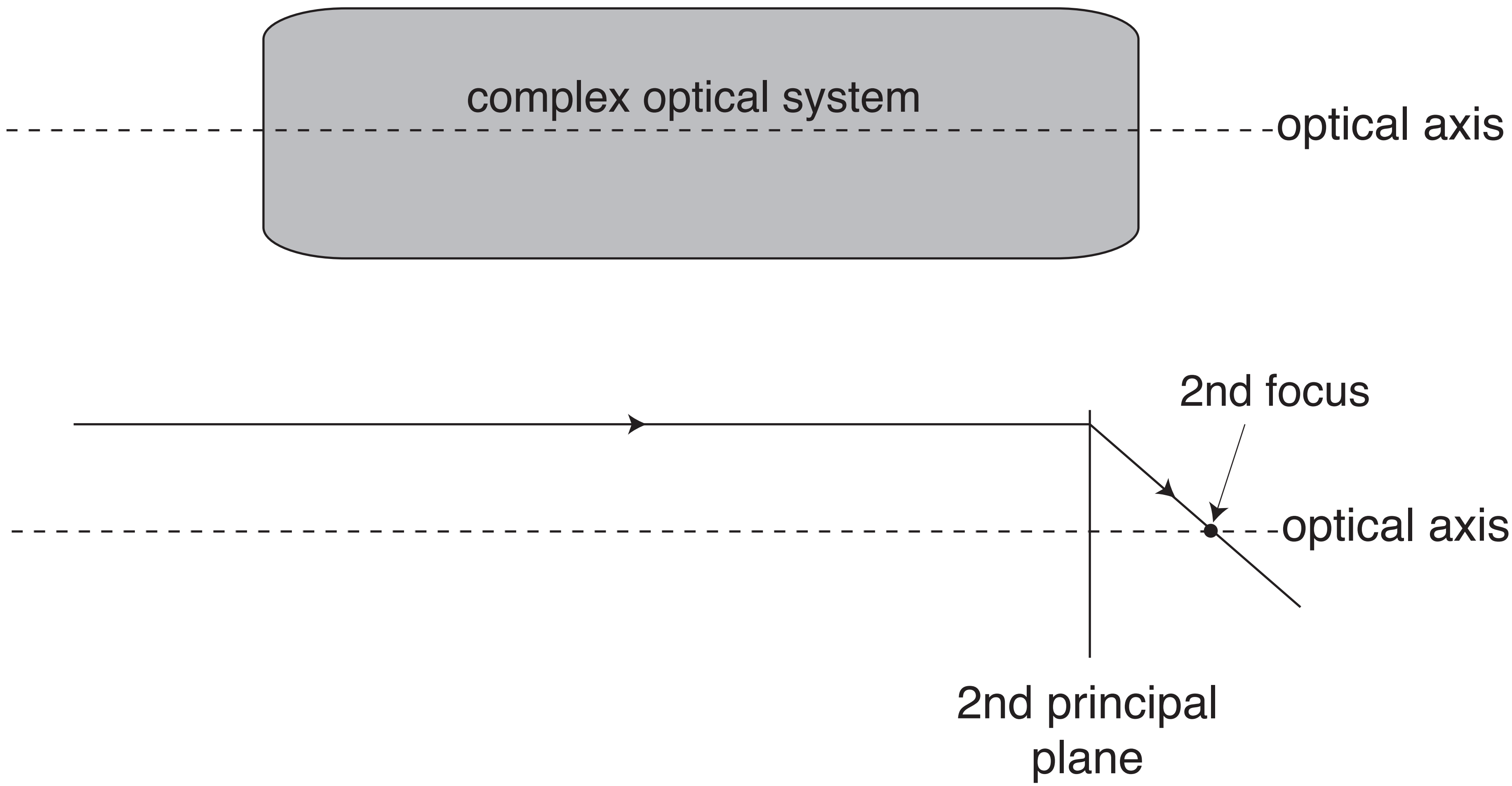


FFL = Front Focal Length

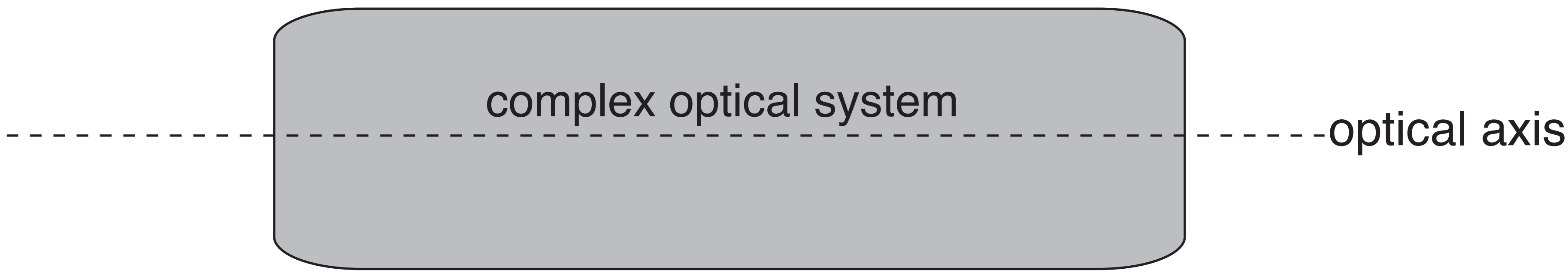
BFL = Back Focal Length

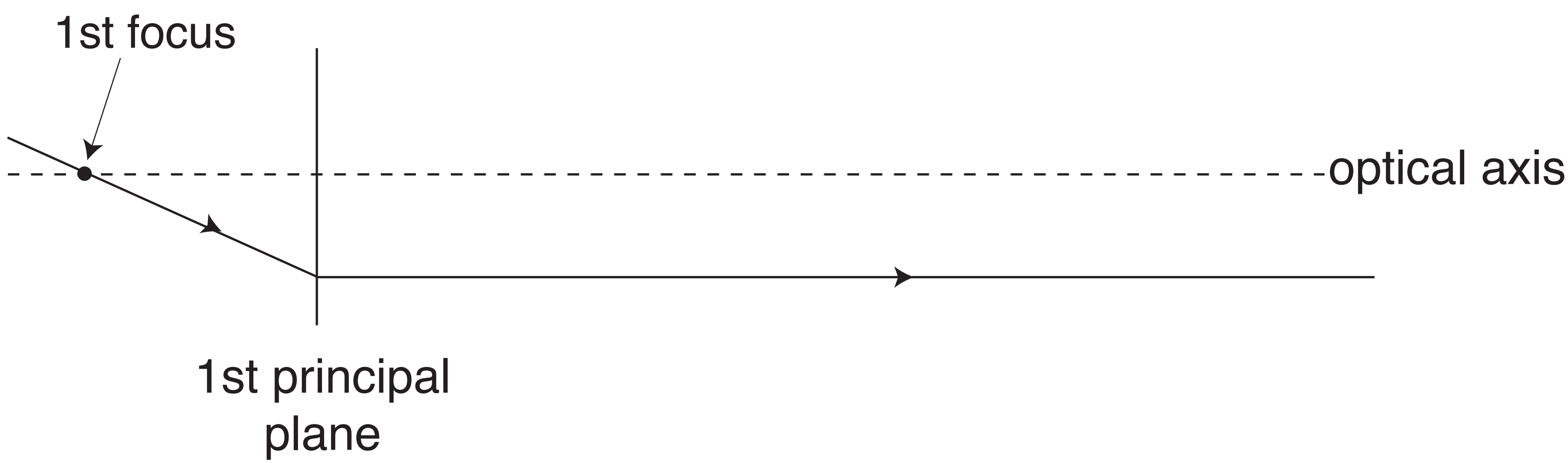
EFL =Effective Focal Length

Significance of Principal Planes

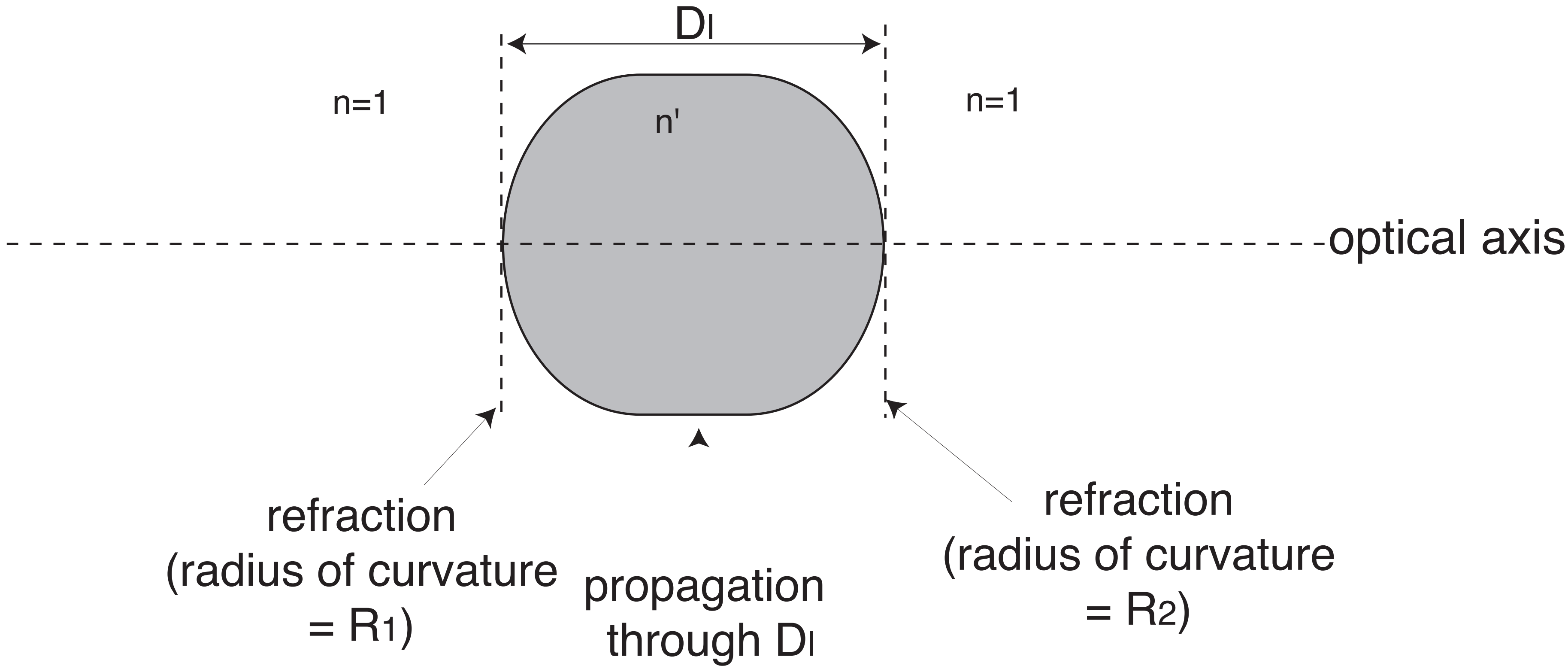


Significance of Principal Planes



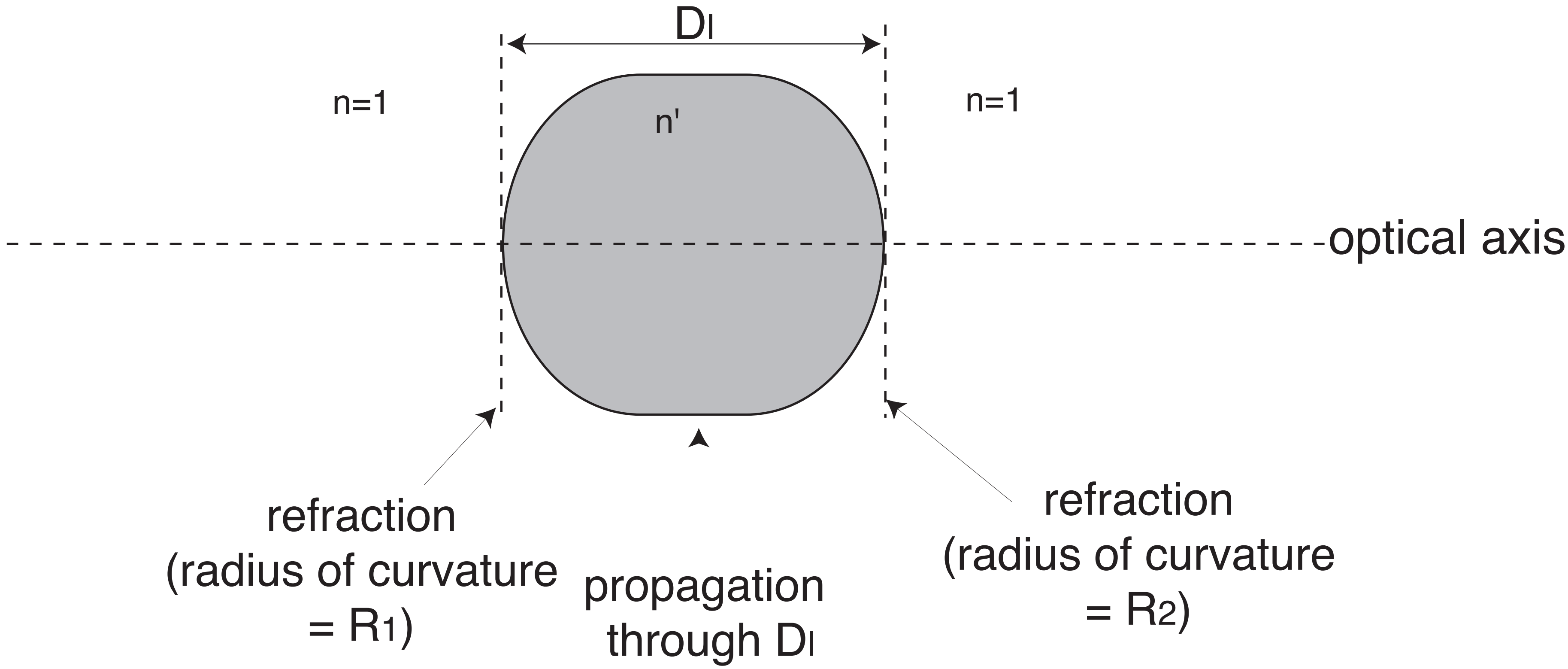


Thick Lens: Matrix Transformation



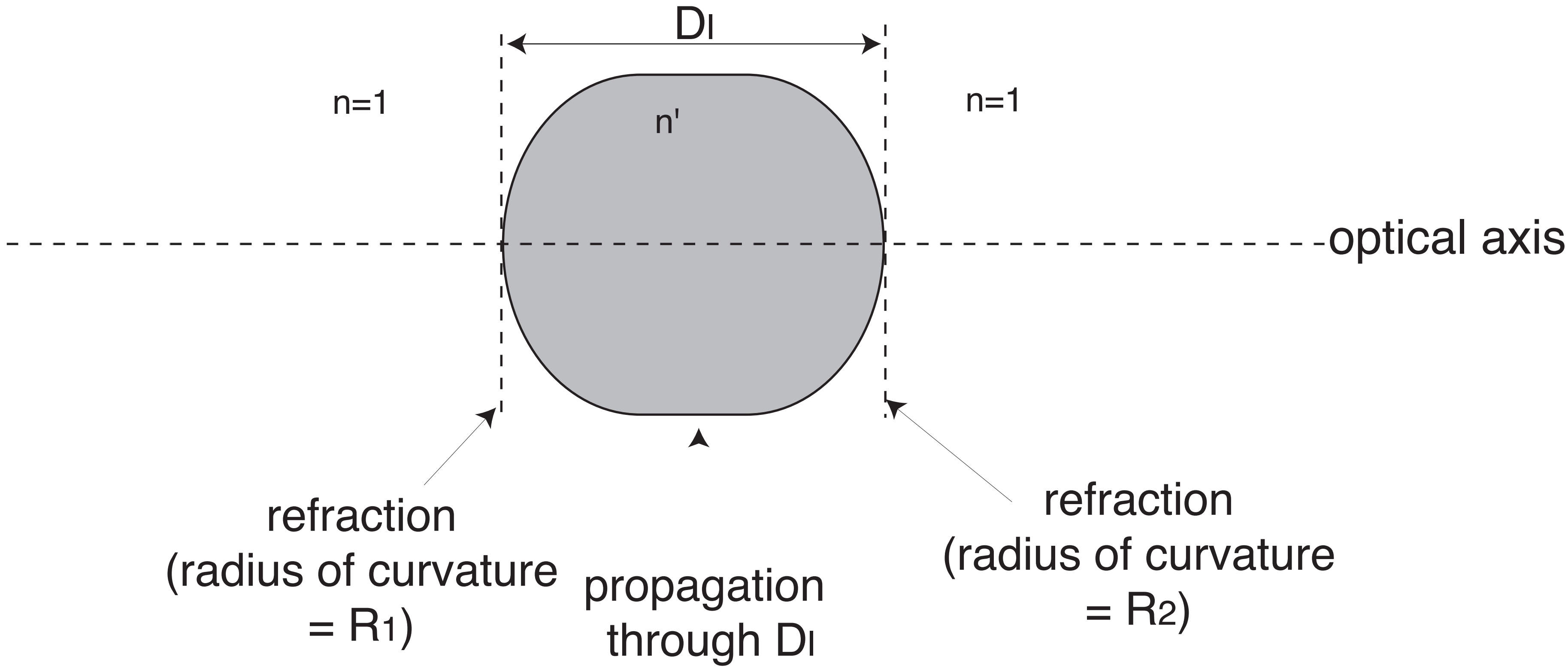
$$\begin{bmatrix} \alpha'_{out} \\ x'_{out} \end{bmatrix} = \begin{bmatrix} 1 & -\left(\frac{1 - n'}{R_2}\right) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{D_l}{n'} & 1 \end{bmatrix} \begin{bmatrix} 1 & -\left(\frac{n' - 1}{R_1}\right) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_{in} \\ x_{in} \end{bmatrix}$$

Thick Lens: Matrix Transformation



$$\begin{bmatrix} \alpha'_{out} \\ x'_{out} \end{bmatrix} = \begin{bmatrix} 1 + \dfrac{D_l}{n'}\left(\dfrac{n'-1}{R_2}\right) & -(n'-1)\left\{ \dfrac{1}{R_1} - \dfrac{1}{R_2} + (n'-1)\dfrac{D_l}{n'R_1R_2} \right\} \\ \dfrac{D_l}{n'} & 1 - \dfrac{D_l}{n'}\left(\dfrac{n'-1}{R_1}\right) \end{bmatrix} \begin{bmatrix} \alpha_{in} \\ x_{in} \end{bmatrix}$$

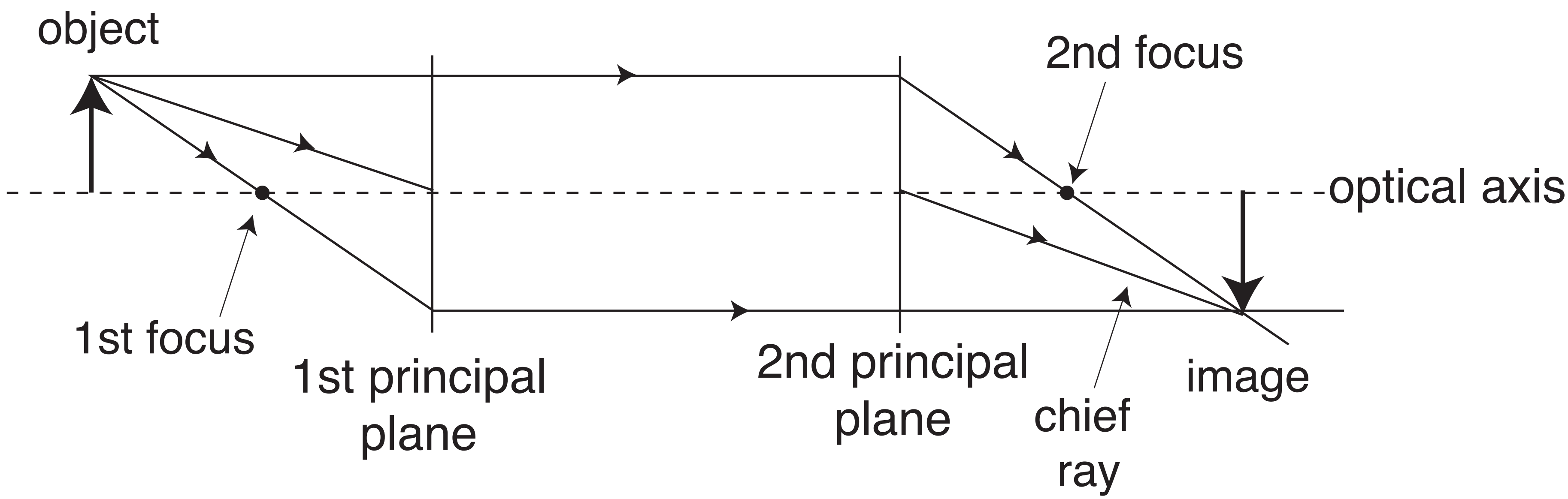
Thick Lens: Matrix Transformation



EFL = f

$$\frac{1}{f} = (n' - 1)\left\{ \frac{1}{R_1} - \frac{1}{R_2} + (n' - 1)\frac{D_l}{n'R_1R_2} \right\}$$

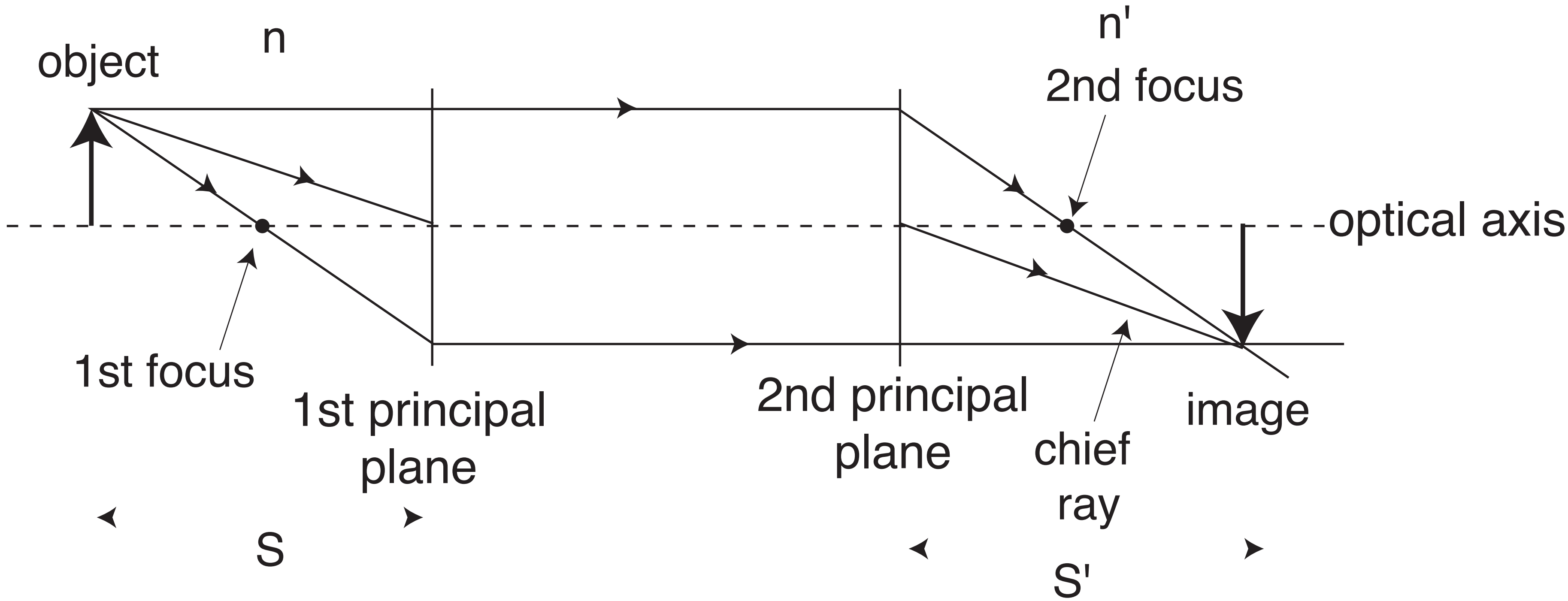
Imaging Condition: Ray Tracing



- Image point is located at the common intersection of all rays emanating from the corresponding object point.

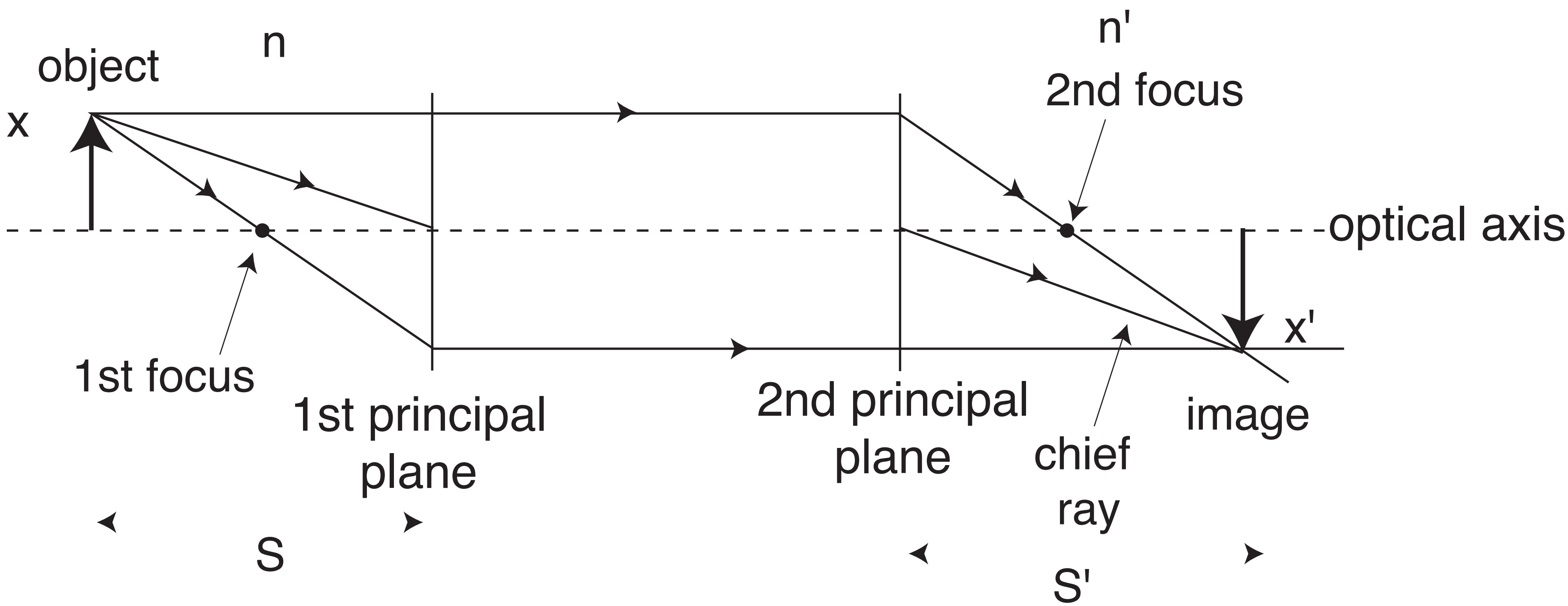
- Rays passing through the two focal points (focii), and the *chief ray* can be ray-traced directly.

Imaging Condition: Matrix Form



system matrix
$$\begin{bmatrix} 1 & 0 \\ S'/n' & 1 \end{bmatrix} \begin{bmatrix} 1 & -P \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ S/n & 1 \end{bmatrix} = \begin{bmatrix} 1 - PS/n & -P \\ S'/n' + S/n - PSS'/nn' & 1 - PS'/n' \end{bmatrix}$$

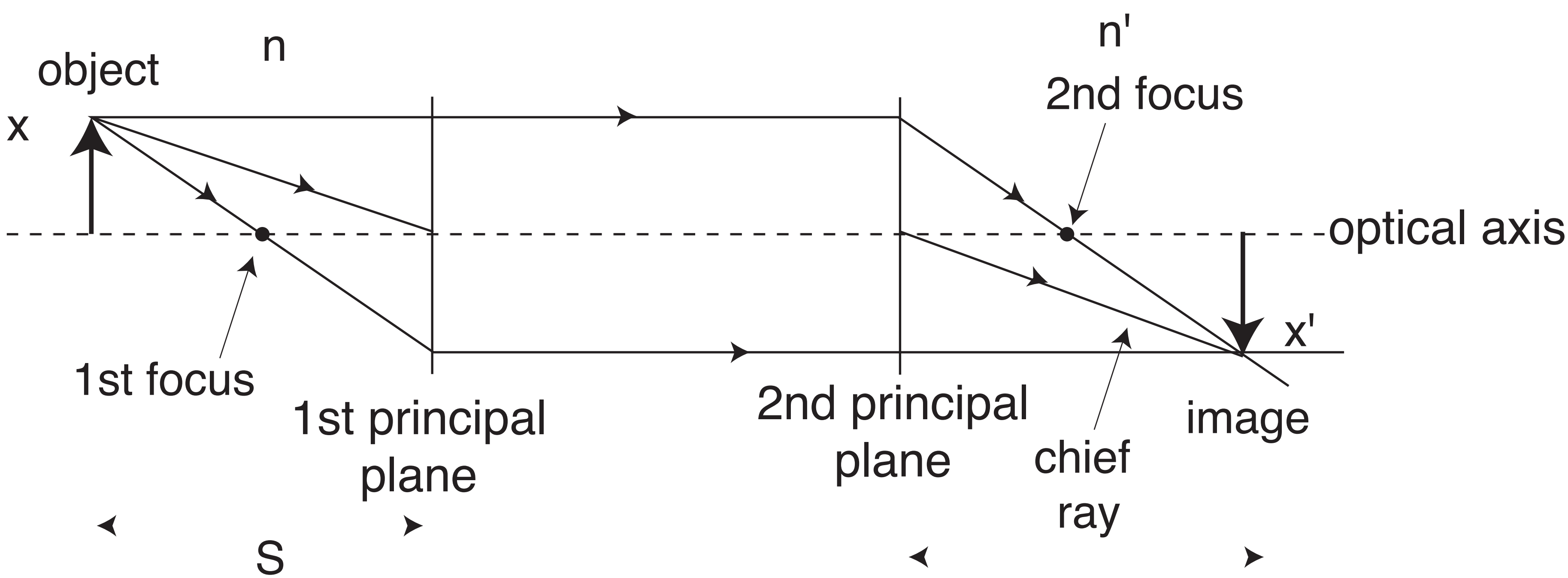
Imaging Condition: Matrix Form



$$\begin{bmatrix} n\alpha' \\ x' \end{bmatrix} = \begin{bmatrix} 1 - PS/n & -P \\ S'/n' + S/n - PSS'/nn' & 1 - PS'/n' \end{bmatrix} \begin{bmatrix} n\alpha \\ x \end{bmatrix}$$

$\rightarrow 0$ x' must be independent of α

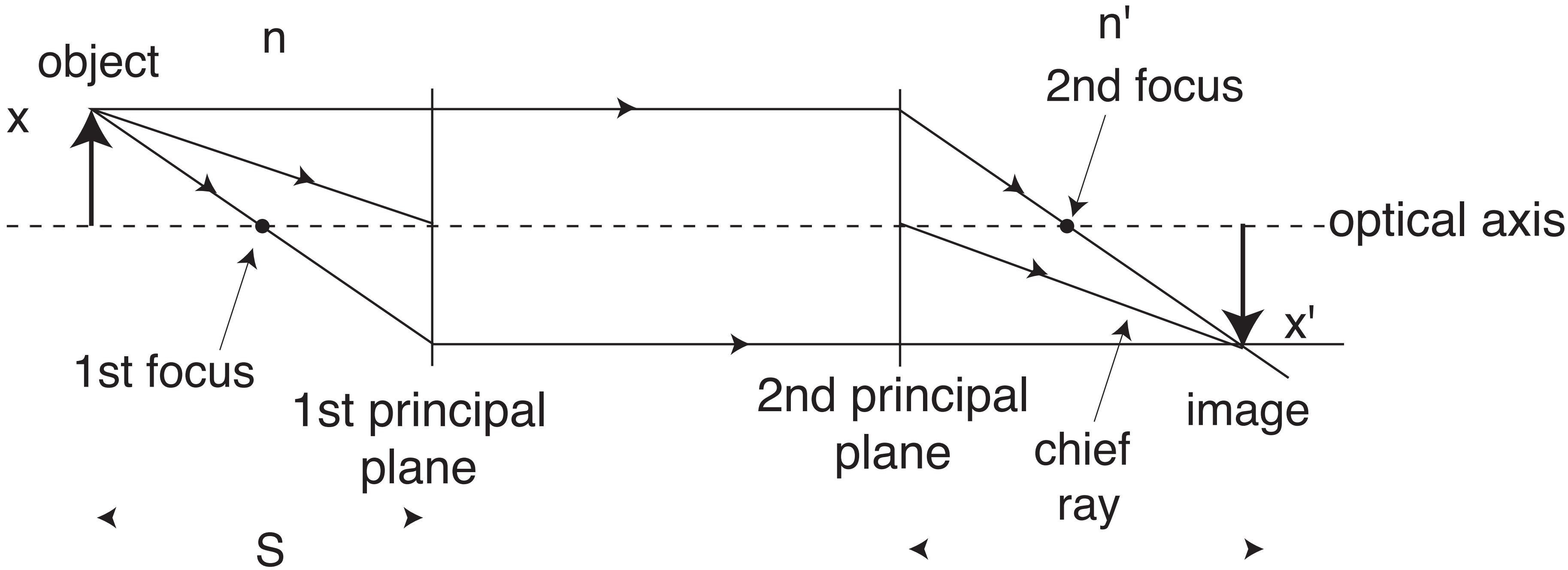
Imaging Condition: Matrix Form



Imaging Condition $$\frac{n}{S} + \frac{n'}{S'} = P$$

System immersed in air $$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f}$$ f = EFL

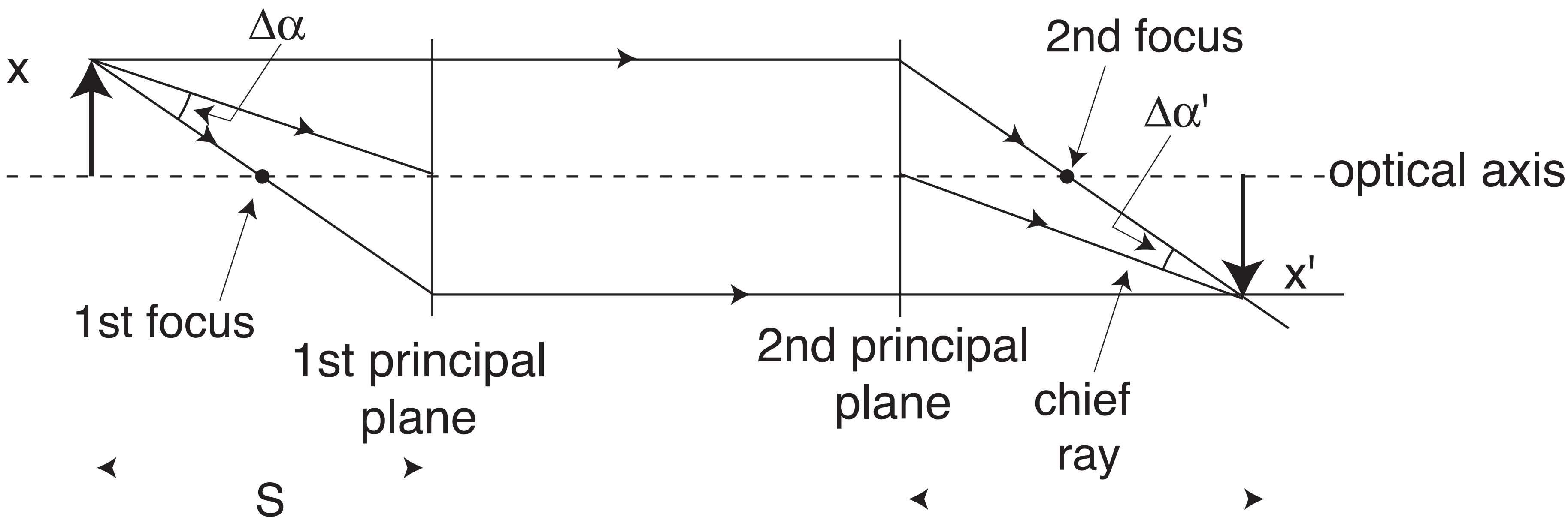
Lateral Magnification



When Imaging Condition is satisfied,

$$m_x = \frac{x'}{x} = 1 - PS'/n'$$

Angular Magnification



When Imaging Condition is satisfied,

$$m_a = \frac{\Delta\alpha'}{\Delta\alpha} = n/n'(1 - PS'/n')$$

Generalized Imaging Conditions

$$\begin{bmatrix} n\,\alpha' \\ x' \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} n\,\alpha \\ x \end{bmatrix}$$

image   system matrix   object

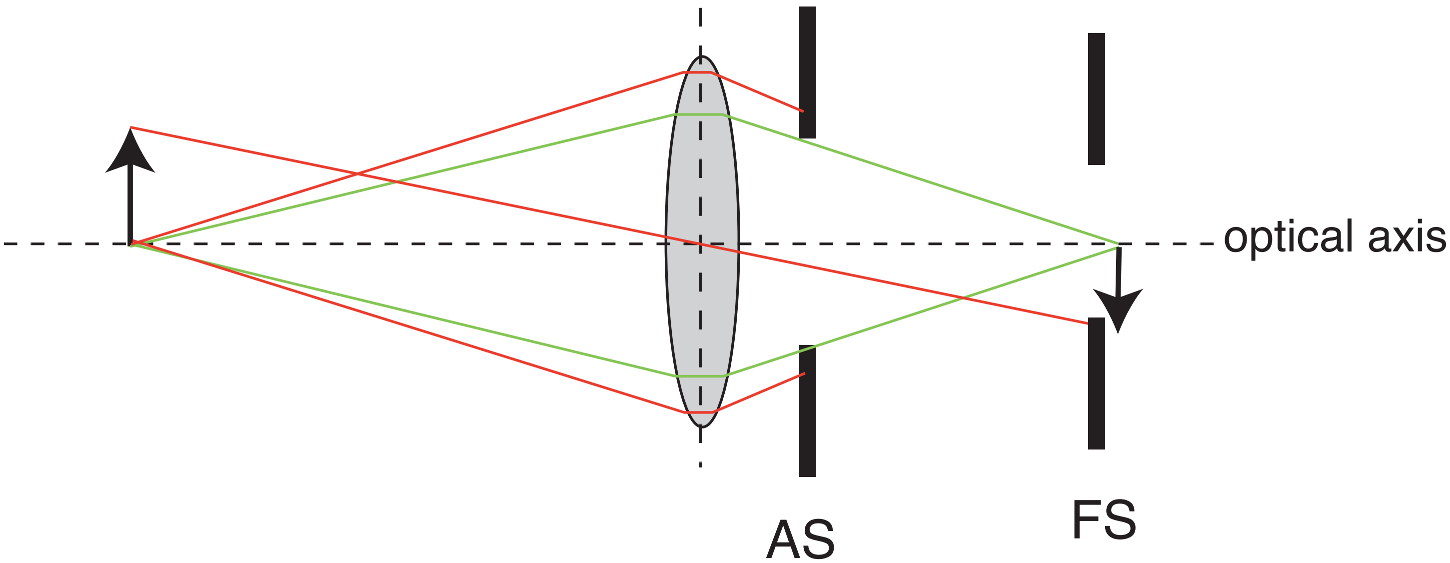
Power $P = -M_{12}$

Imaging Condition $M_{21} = 0$

Lateral Magnification $m_x = M_{22}$

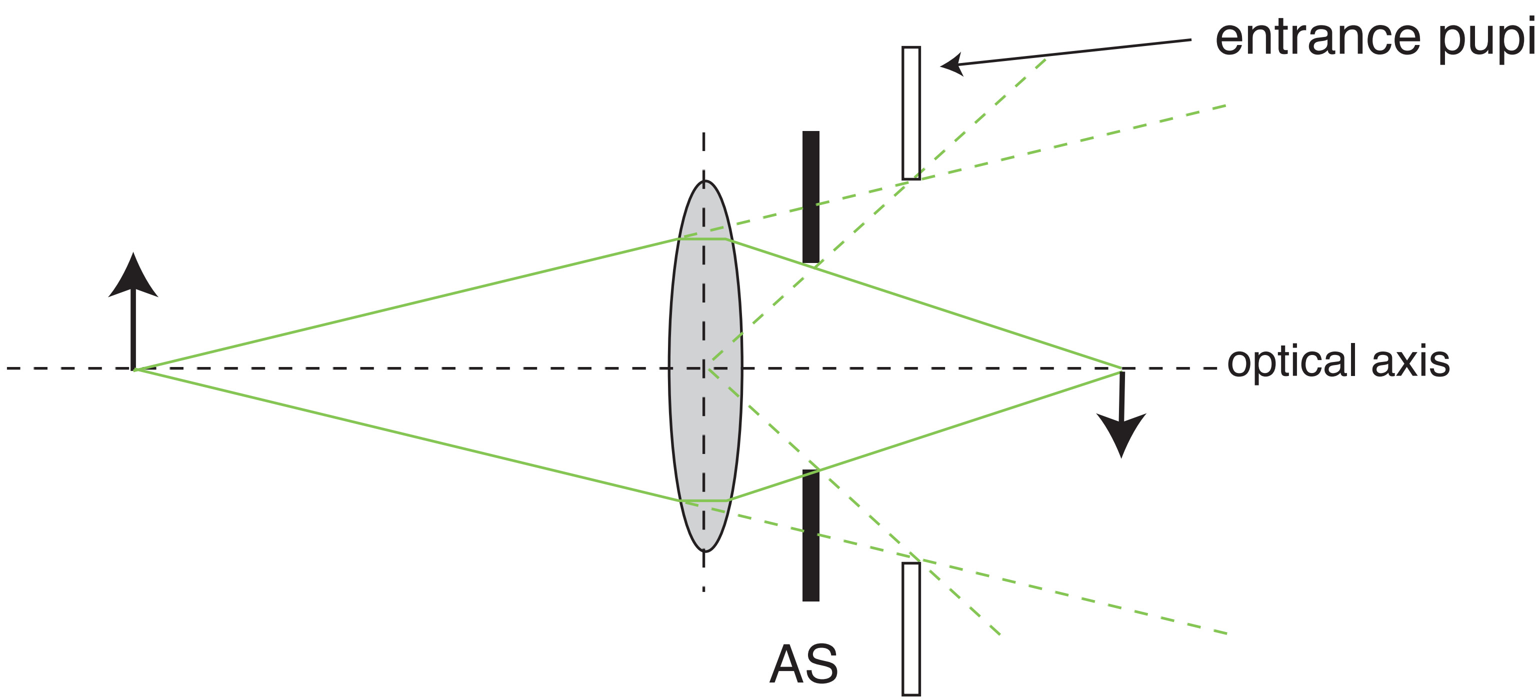
Angular Magnification $m_a = (n/n')M_{11}$

Aperture Stop & Field Stop



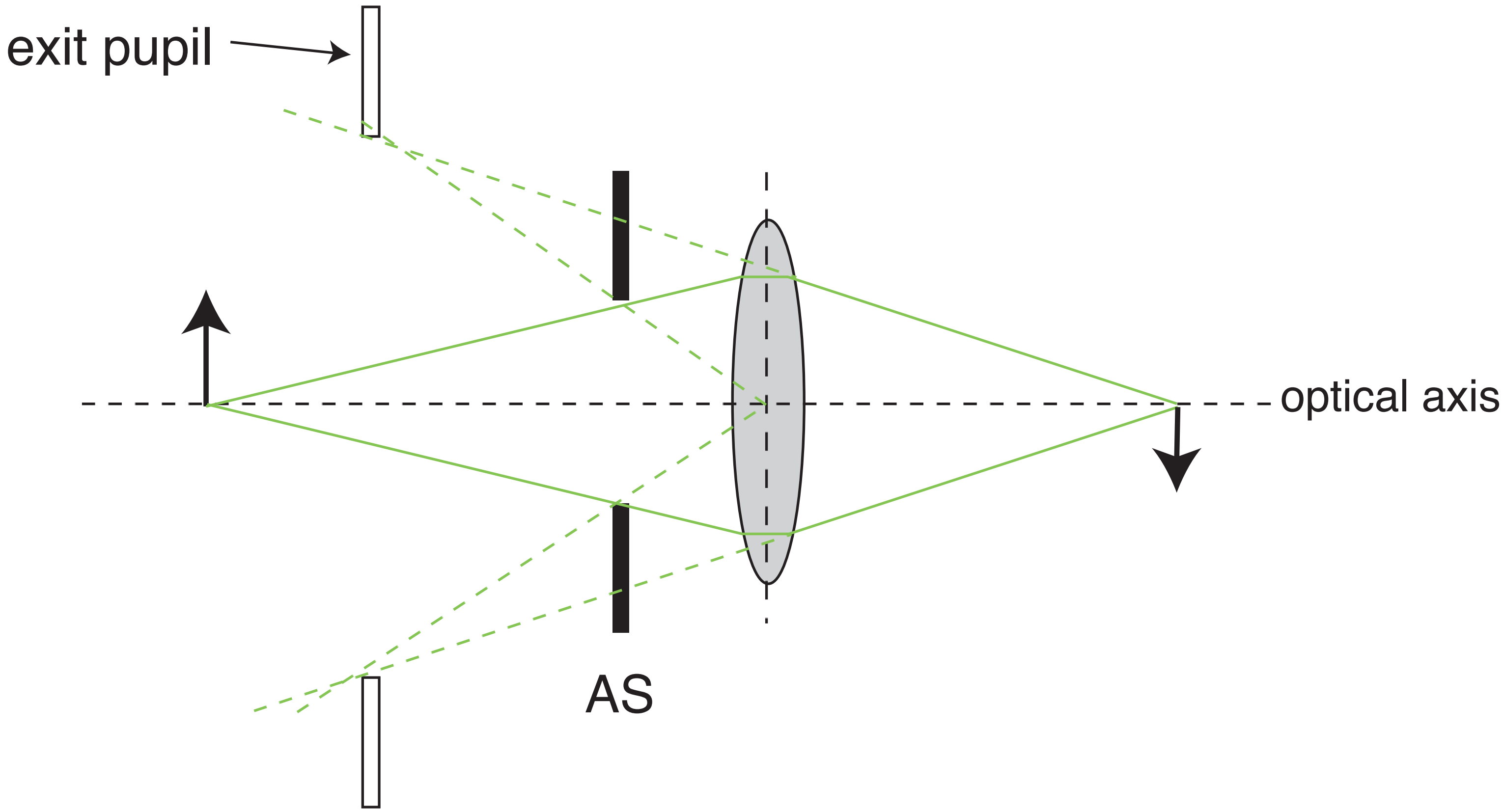
- AS limits the amount of light energy reaching each image point
- FS limits the number of image points (extent of the image)

Entrance Pupil



- Entrance Pupil is the image of the AS seen from the object side.

Exit Pupil

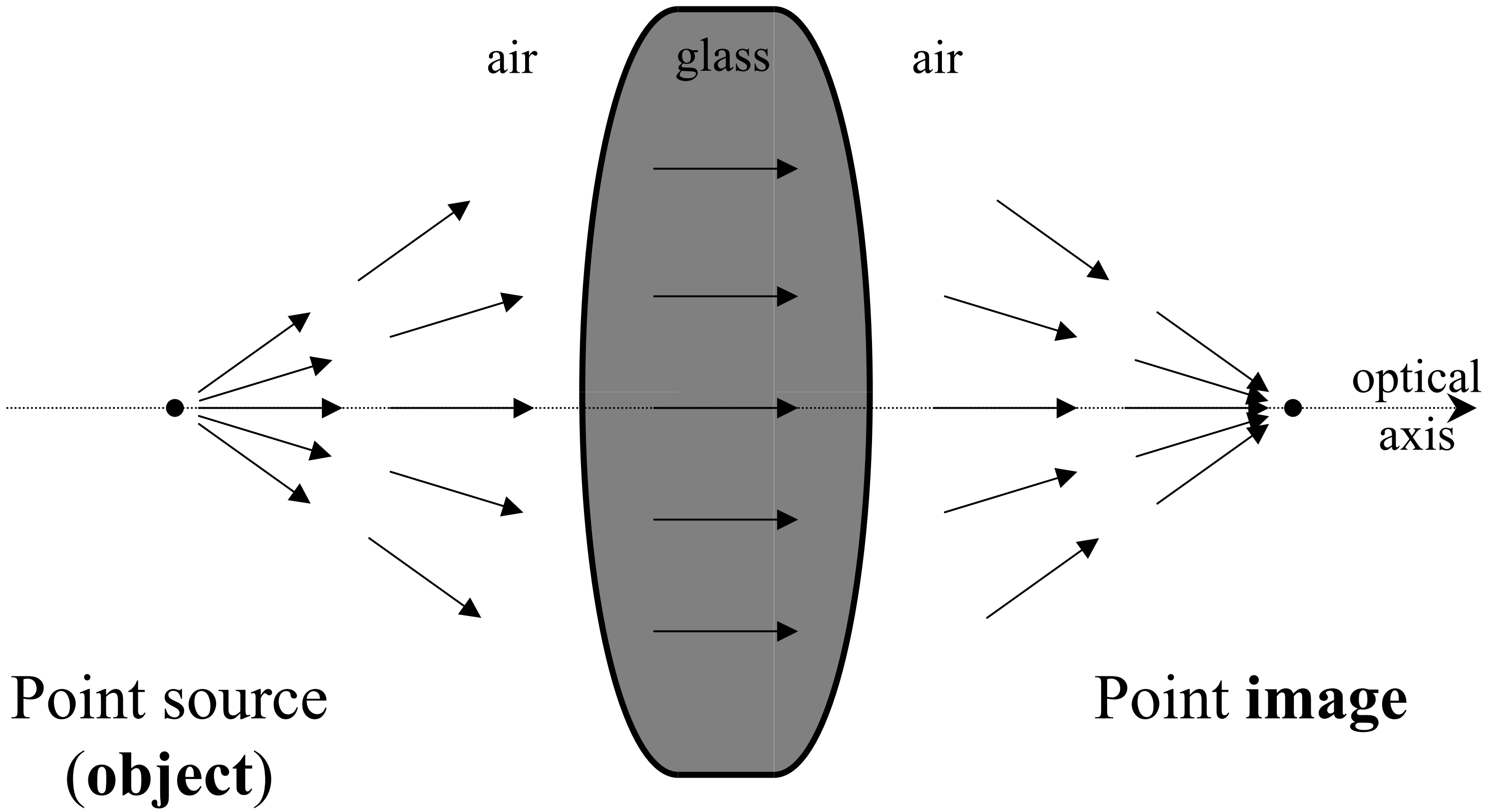


- Exit Pupil is the image of the AS seen from the image side.

Mirrors & prisms

- Last time: optical elements,
  - Lenses
    - Basic properties of spherical surfaces
    - Ray tracing
    - Image formation
    - Magnification
- Today: more optical elements,
  - Prisms
  - Mirrors

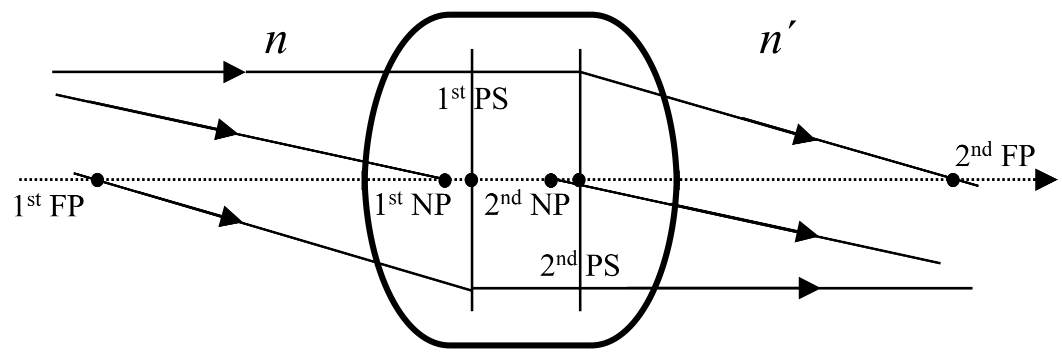
Lens: main instrument for image formation



The curved surface makes the rays bend proportionally to their distance from the "optical axis", according to Snell's law. Therefore, the divergent wavefront becomes convergent at the right-hand (output) side.

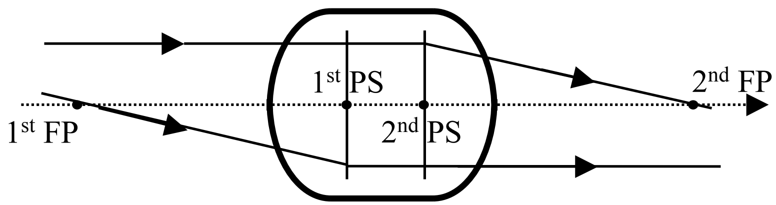
Cardinal Planes and Points

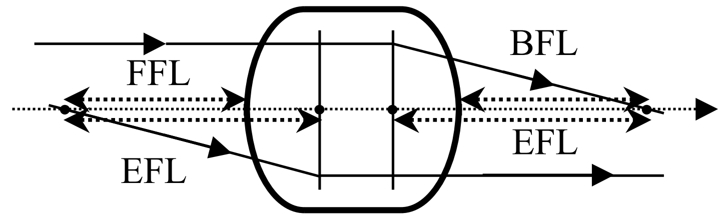
- Rays generated from axial point at infinity (*i.e*., forming a ray bundle parallel to the optical axis) and entering an optical system intersect the optical axis at the Focal Points.
- The intersection of the extended entering parallel rays and the extended exiting convergent rays forms the Principal Surface (Plane in the paraxial approximation.)
- The extension of a ray which enters and exits the optical system with the same angle of propagation intersects the optical axis at the Nodal Points.



Recap of lens-like instruments

- Cardinal Points and Focal Lengths





$$\begin{pmatrix} n'\alpha' \\ x' \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} n\alpha \\ x \end{pmatrix}$$

Matrix formulation

- Imaging conditions

$M_{12} \neq 0$

$P = -M_{12} \neq 0$

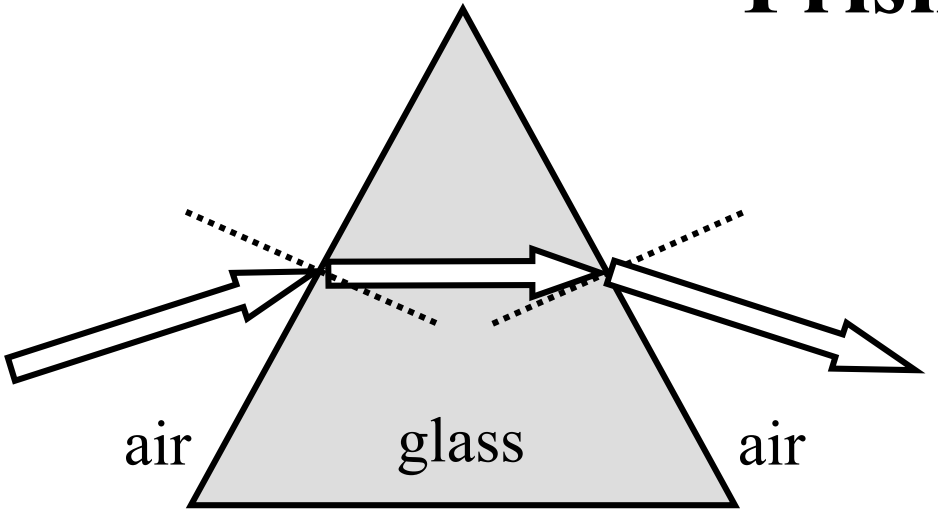
$M_{21} = 0$

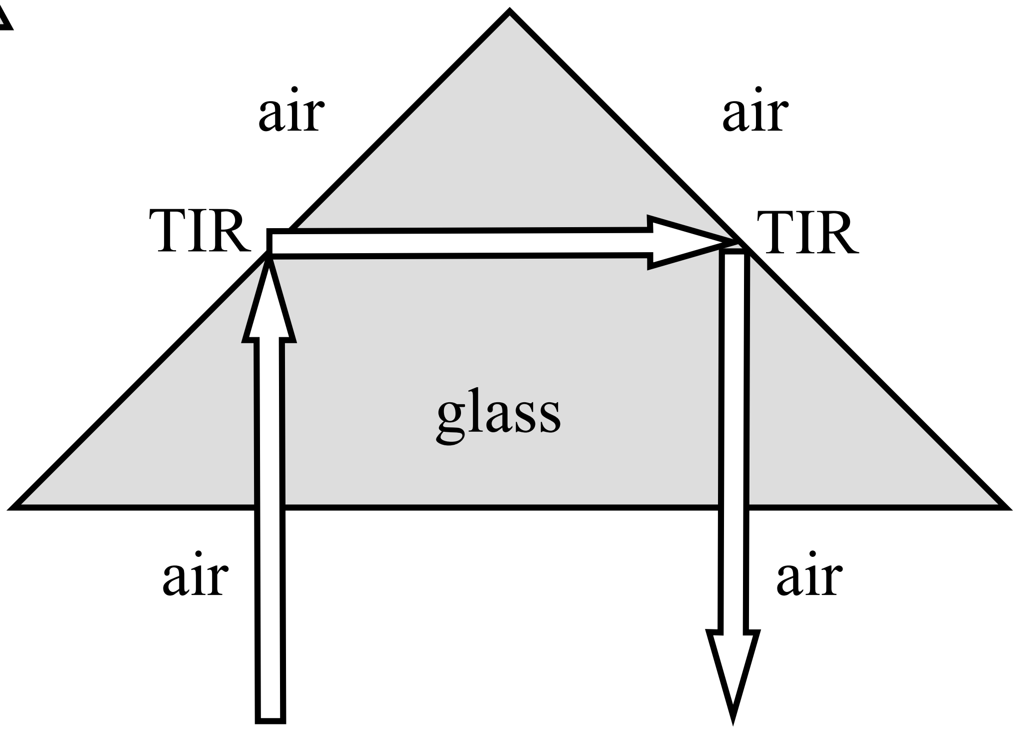
Magnification

lateral $m_x = M_{22}$

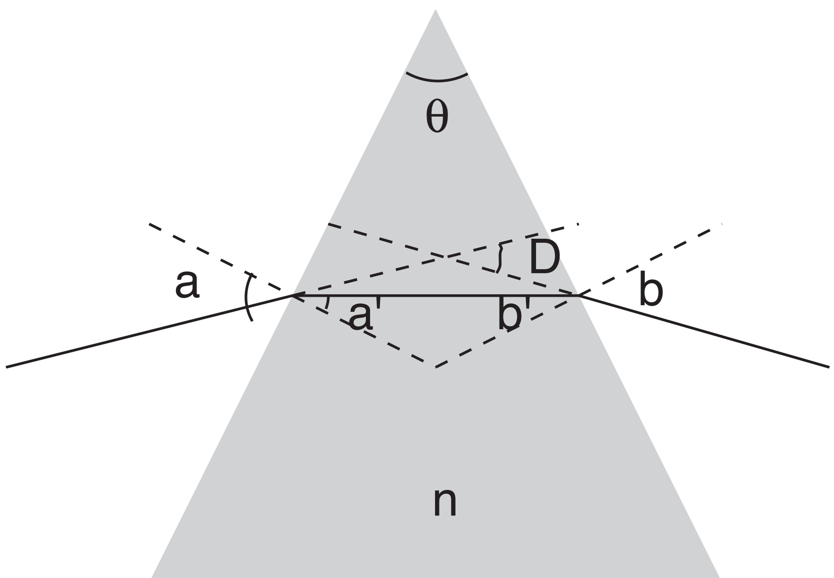
angular $m_a = \dfrac{n}{n'} M_{11}$

Prisms





Refracting Prism



Assume a symmetric case,

a = b
a' = b'

$$a' = \frac{\theta}{2}$$

$$a = \frac{\theta + D}{2}$$

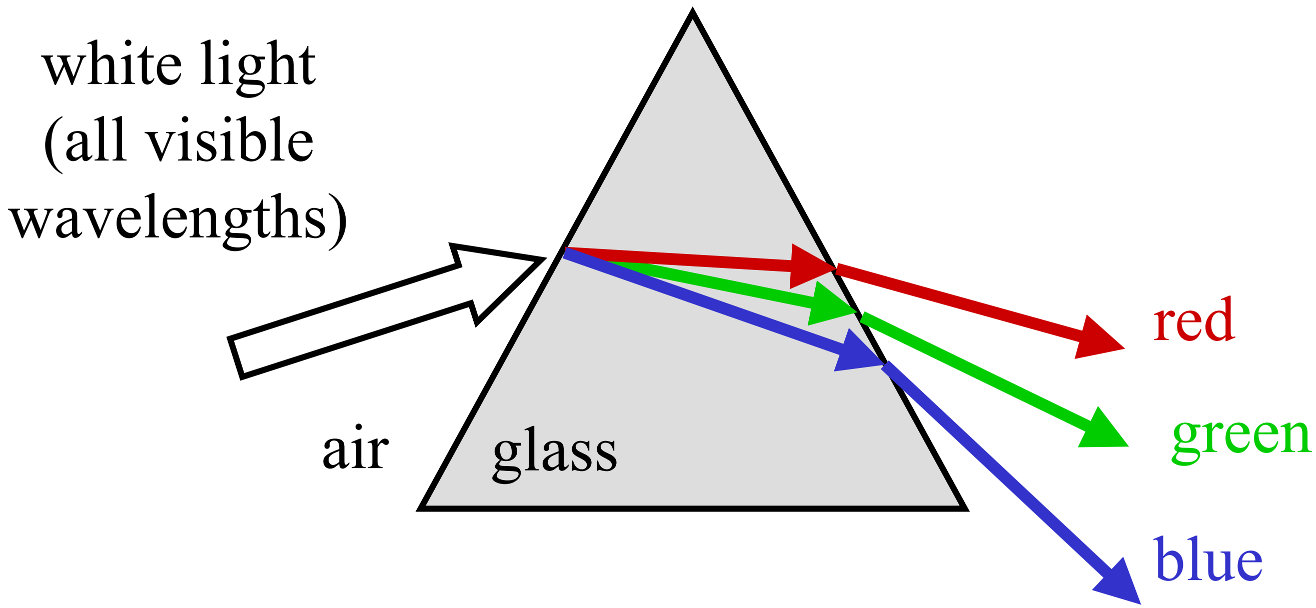
From Snell's law,

Prism Equation

$$n = \frac{\sin(\frac{D+\theta}{2})}{\sin(\frac{\theta}{2})}$$

Dispersion

Refractive index *n* is function of the wavelength



Newton's prism

Dispersion measures

Reference color lines
C (H- λ=656.3nm, red), D (Na- λ=589.2nm, yellow),
F (H- λ=486.1nm, blue)

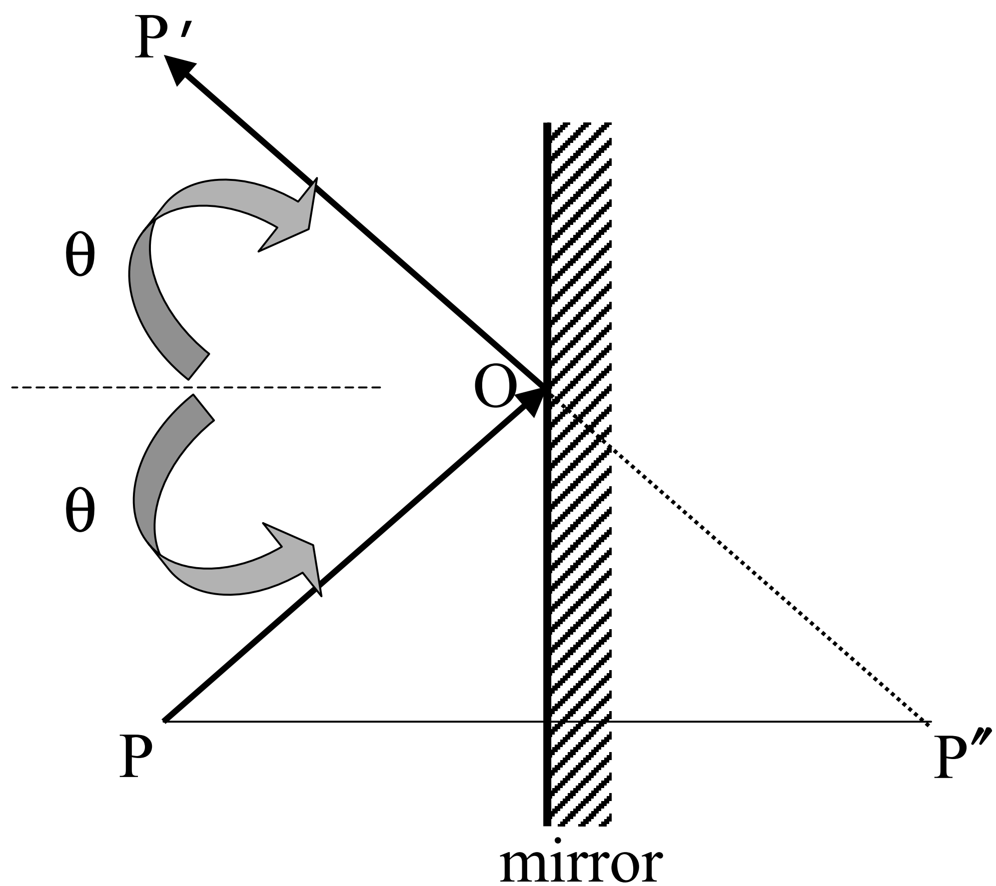
Crown glass has

$n_{\mathrm{F}} = 1.52933 \qquad n_{\mathrm{D}} = 1.52300 \qquad n_{\mathrm{C}} = 1.52042$

Dispersive power $V = \dfrac{n_{\mathrm{F}} - n_{\mathrm{C}}}{n_{\mathrm{D}} - 1}$

Dispersive index $v = \dfrac{1}{V} = \dfrac{n_{\mathrm{D}} - 1}{n_{\mathrm{F}} - n_{\mathrm{C}}}$

Mirrors: the law of reflection



Plane Mirrors

Plane Mirrors have zero power.

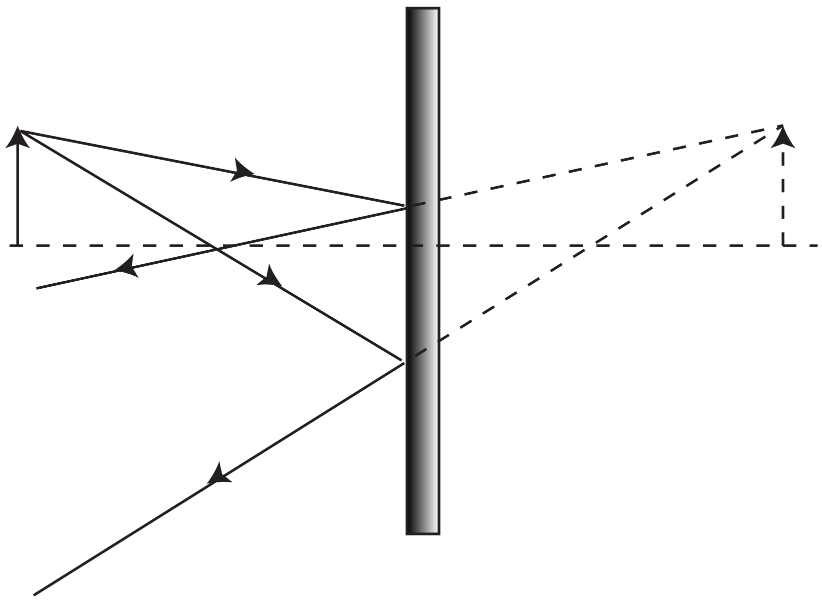
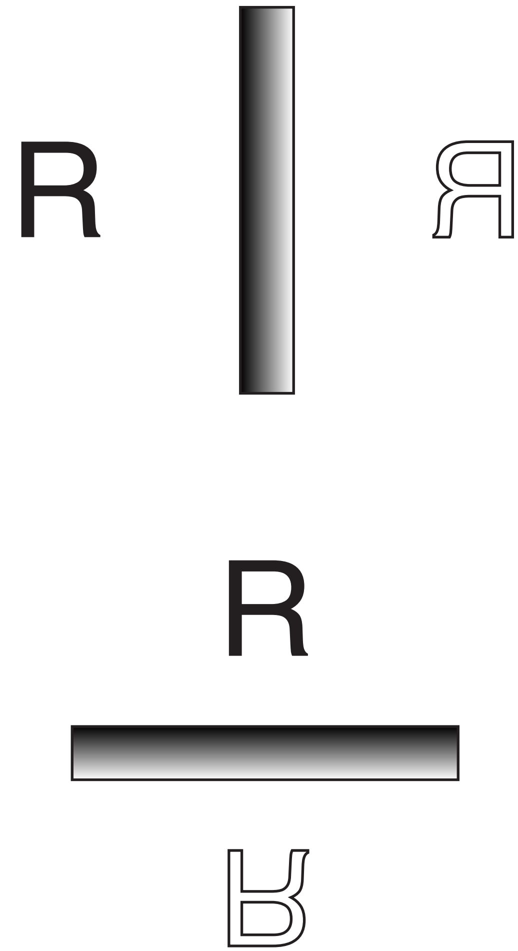


Image is always virtual

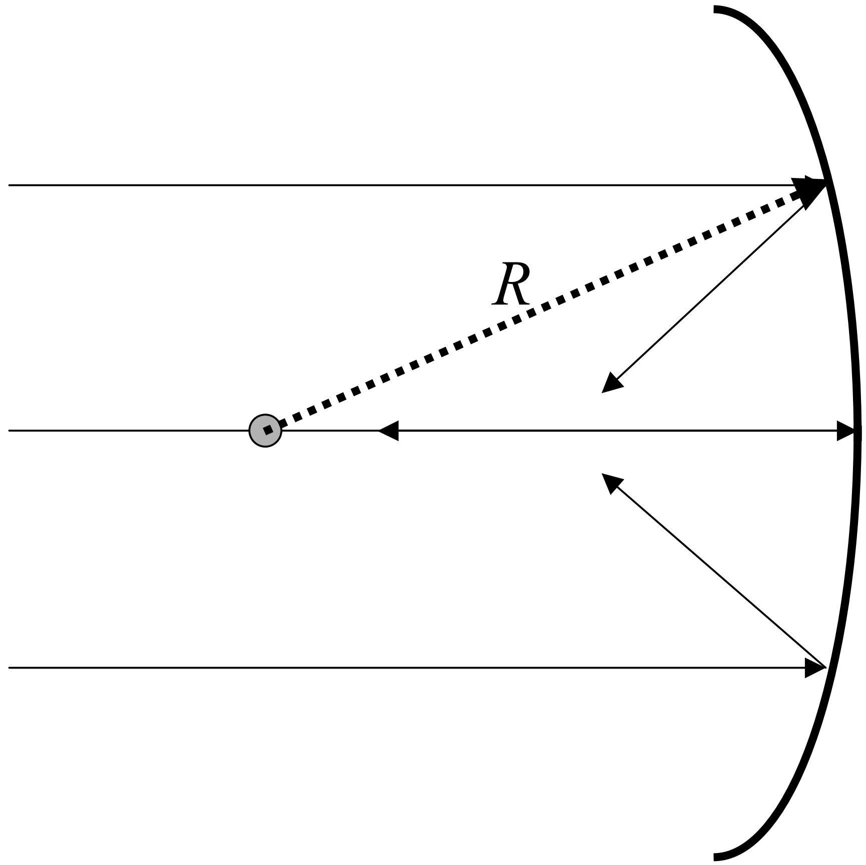
Images are laterally inverted



Sign conventions for reflection

- Light travels from left to right *before reflection and from right to left after reflection*
- A radius of curvature is positive if the surface is convex towards the left
- Longitudinal distances *before reflection* are positive if pointing to the right; *longitudinal distances after reflection are positive if pointing to the left*
- Longitudinal distances are positive if pointing up
- Ray angles are positive if the ray direction is obtained by rotating the +z axis counterclockwise through an acute angle

Example: spherical mirror



In the paraxial approximation,
It (approximately) focuses an
Incoming parallel ray bundle
(from infinity) to a point.

Reflective optics formulae

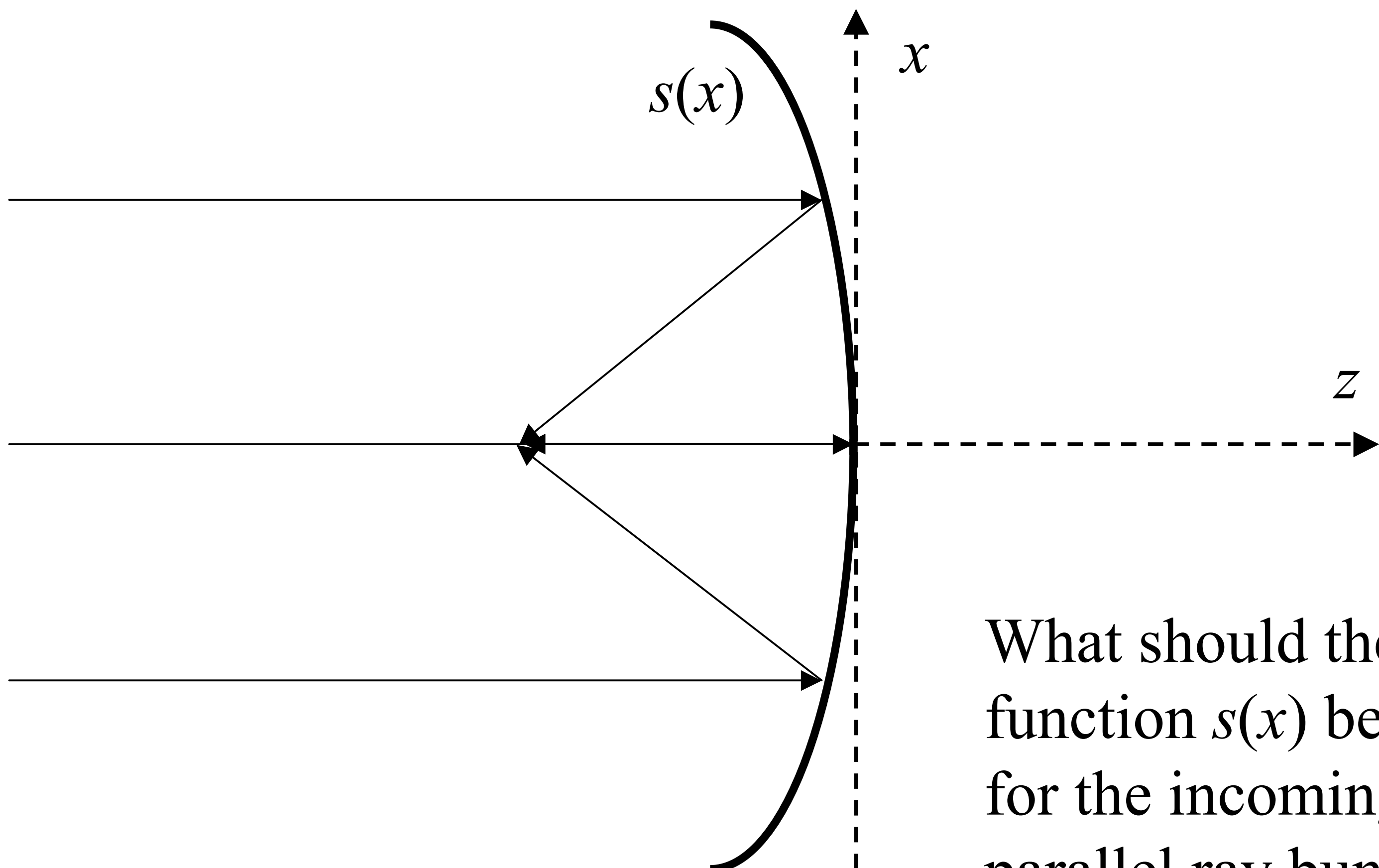
Imaging condition $$\frac{1}{D_{12}}+\frac{1}{D_{01}}=-\frac{2}{R}$$

Focal length $$f=-\frac{R}{2}$$

Magnification $$m_x=-\frac{D_{12}}{D_{01}} \qquad m_\alpha=-\frac{D_{01}}{D_{12}}$$

Paraboloid mirror: perfect focusing

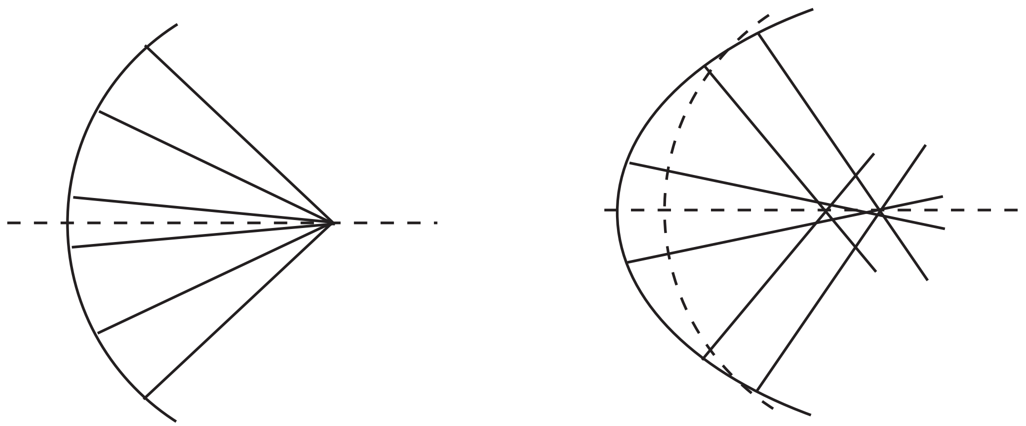
(e.g. satellite dish)



What should the shape function $s(x)$ be in order for the incoming parallel ray bundle to come to perfect focus?

Aberrations

- Deviation of the wavefront from its ideal spherical shape due to the imperfect refraction/reflection by the optical elements.



perfect spherical
wavefront focused
to a point.

aberrated wavefront
does not come to a
focus. Image is blurred.

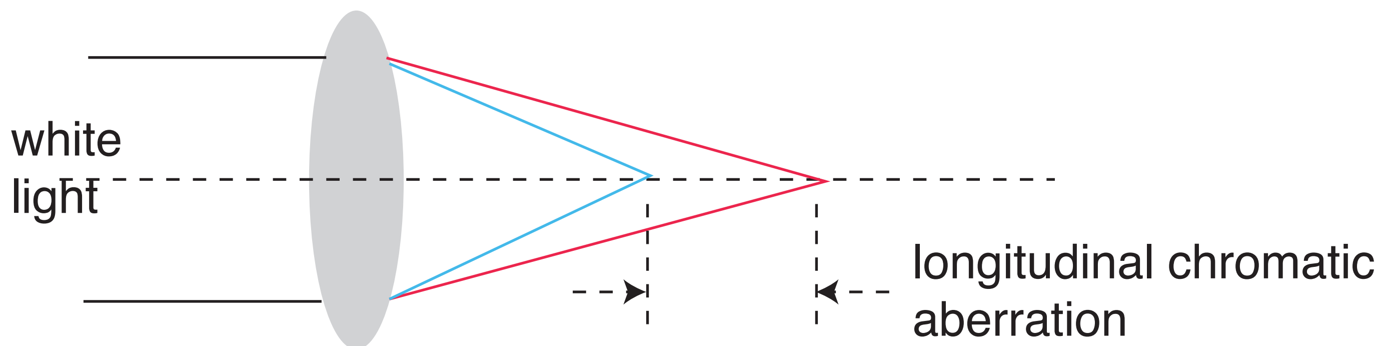
- Optical elements (lenses, mirrors) produce perfect spherical wavefronts only in the paraxial approximation (i.e. for small angles of propagation with respect to the optical axis).

- At larger angles, Seidel (or primary) aberrations occur.

Chromatic Aberration

- Index of refraction varies with wavelenght (index for blue > index for red)

- Blue light comes to focus closer to the lens than red light. The horizontal distance between the two images is called *longitudinal chromatic aberration*.

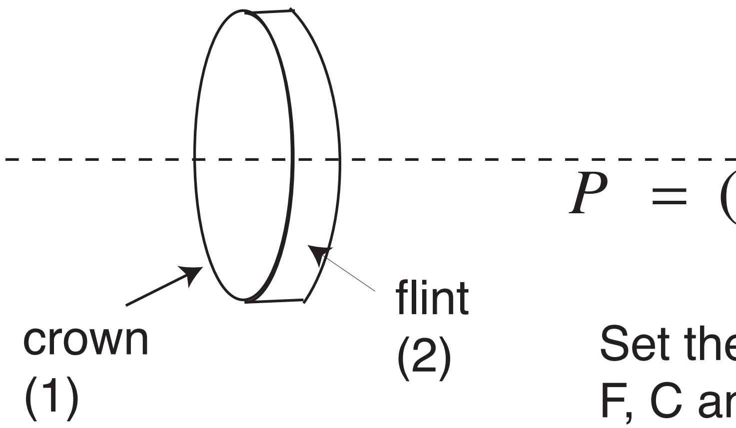


- Images formed by different wavelengths also have different transverse (lateral) magnifications. This is called *lateral chromatic aberration*.

- Image formed by blue light is smaller and closer to the lens than that formed by red light.

Correcting Chromatic Aberration

- Using a combination of two kinds of glasses, crown and flint to form a dichromat. Crown can have more +ve power and has moderate dispersion, while flint can have lower -ve power and has high dispersion.



$$P = P_1 + P_2$$

$$P = (n_1 - 1)(\frac{1}{R_1} - \frac{1}{R_2}) + (n_2 - 1)(\frac{1}{R_1} - \frac{1}{R_2})$$

Set the powers equal for the wavelengths, F, C and e.

$$\frac{P_1}{P_2} = \frac{V_1}{V_2}$$

$$V = \frac{n_e - 1}{n_F - n_C}$$ (dispersion factor or Abbe's factor)

$$P_1 = P\frac{V_1}{V_1 - V_2} \qquad P_2 = -P\frac{V_2}{V_1 - V_2}$$

Summary of Aberrations

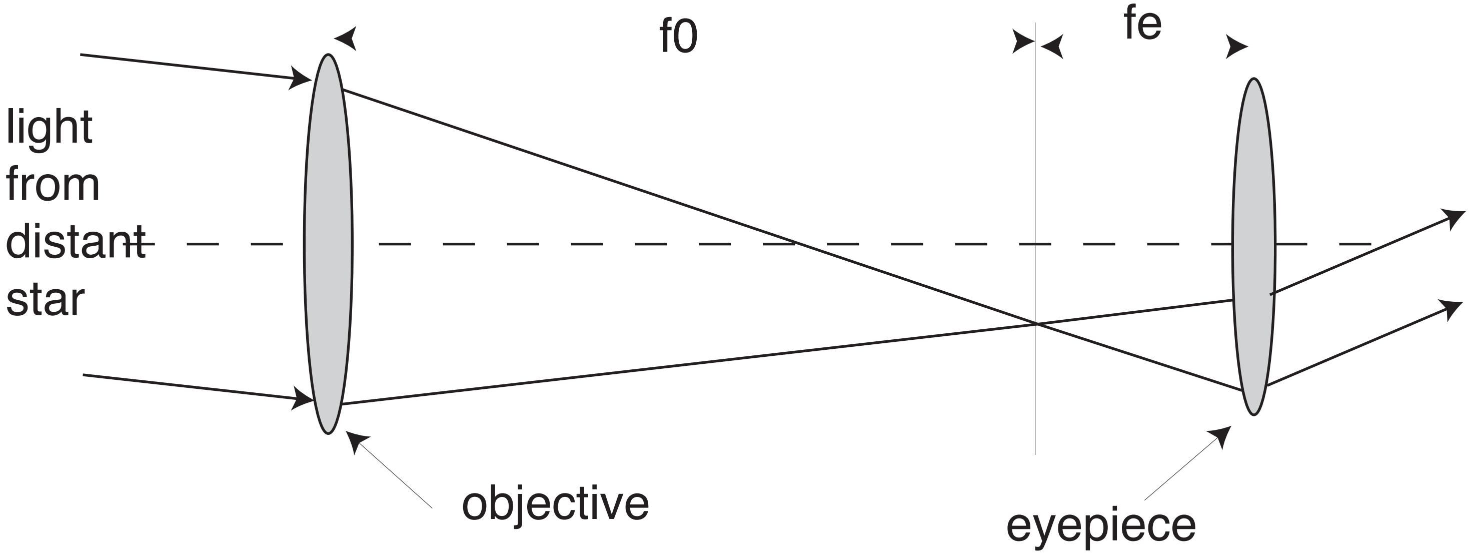
Aberration	Character	Correction
Spherical aberration	monochromatic, on- and off-axis, image blur	Aspherics, doublet, high-index
Coma	monochromatic, off-axis only, blur	spaced doublet with central stop
Oblique astigmatism	monochromatic, off-axis, blur	spaced doublet with stop
Curvature of field	monochromatic, off-axis	spaced doublet
Distortion	monochromatic, off-axis	spaced doublet with stop
Chromatic aberration	polychromatic, on- and off-axis, blur	contact doublet, spaced doublet

Optical Systems

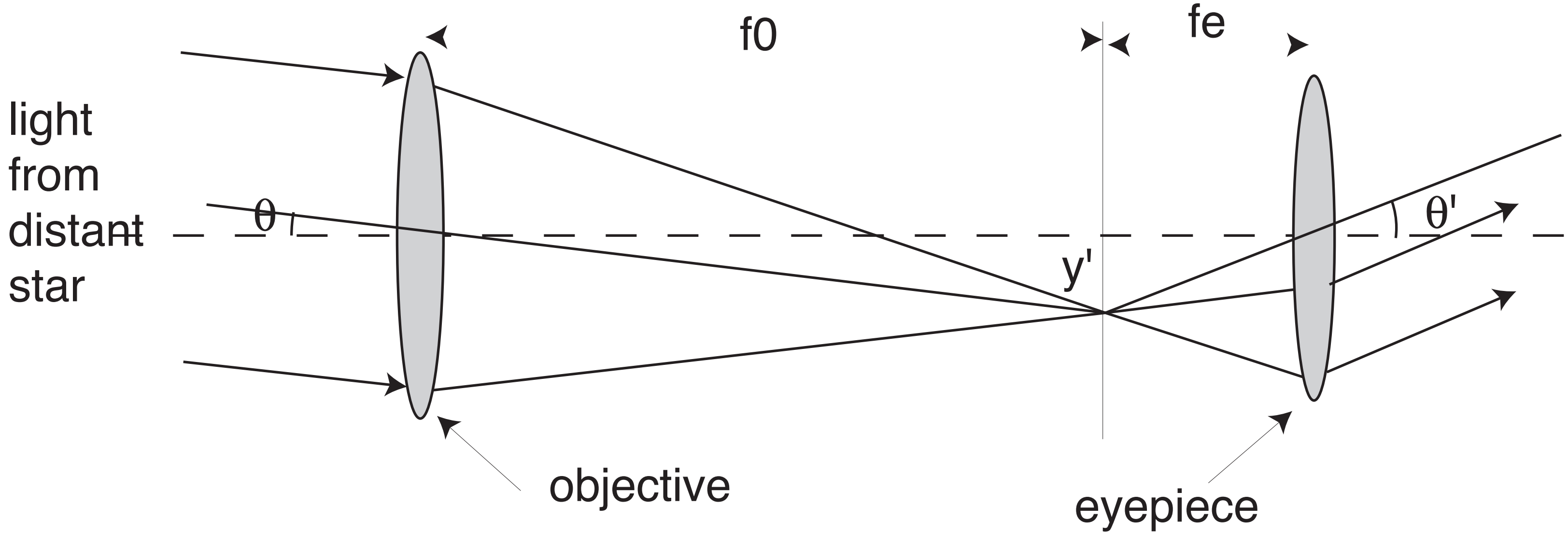
General criterion while designing an optical system:

- Light gathering power (capacity to form a bright image)
- Magnification
- Resolving power (capacity to form sharp images of small detail)
- Others such as physical size, weight, cost etc.

Astronomical Telescope



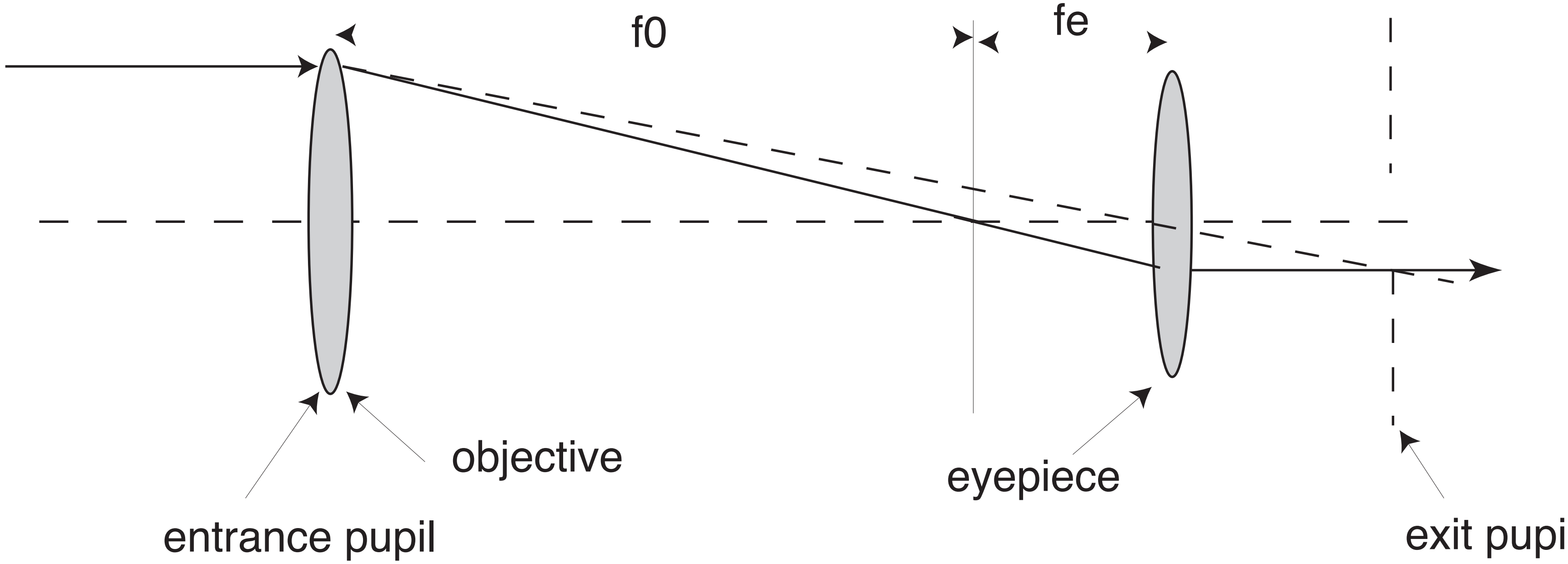
Angular Magnification of Telescope



$$M = \frac{\text{angular size of image}}{\text{angular size of object}} = \frac{\tan\theta'}{\tan\theta}$$

$$= \frac{y'/-fe}{y'/f0} = -\frac{f0}{fe}$$

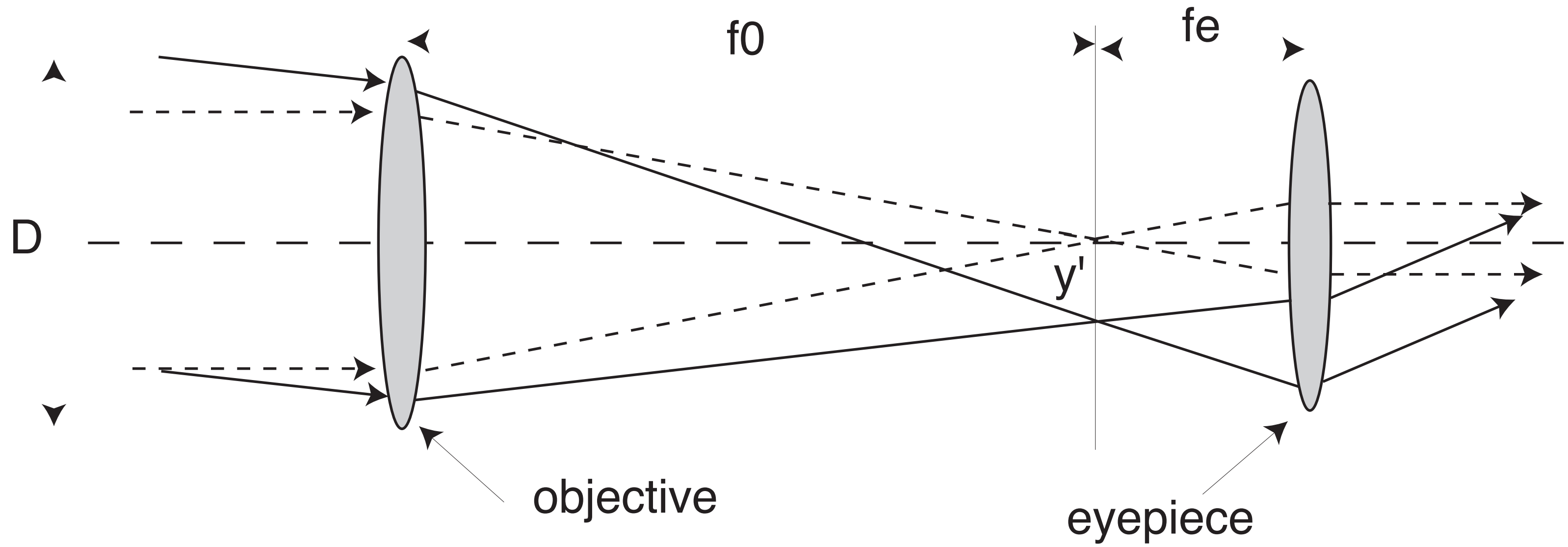
Angular Magnification of Telescope



$$\frac{\text{diameter of entrance pupil}}{f0} = \frac{\text{diameter of exit pupil}}{-fe}$$

$$M = -\frac{f0}{fe} = \frac{\text{diameter of entrance pupil}}{\text{diameter of exit pupil}}$$

Angular Resolution of Telescope

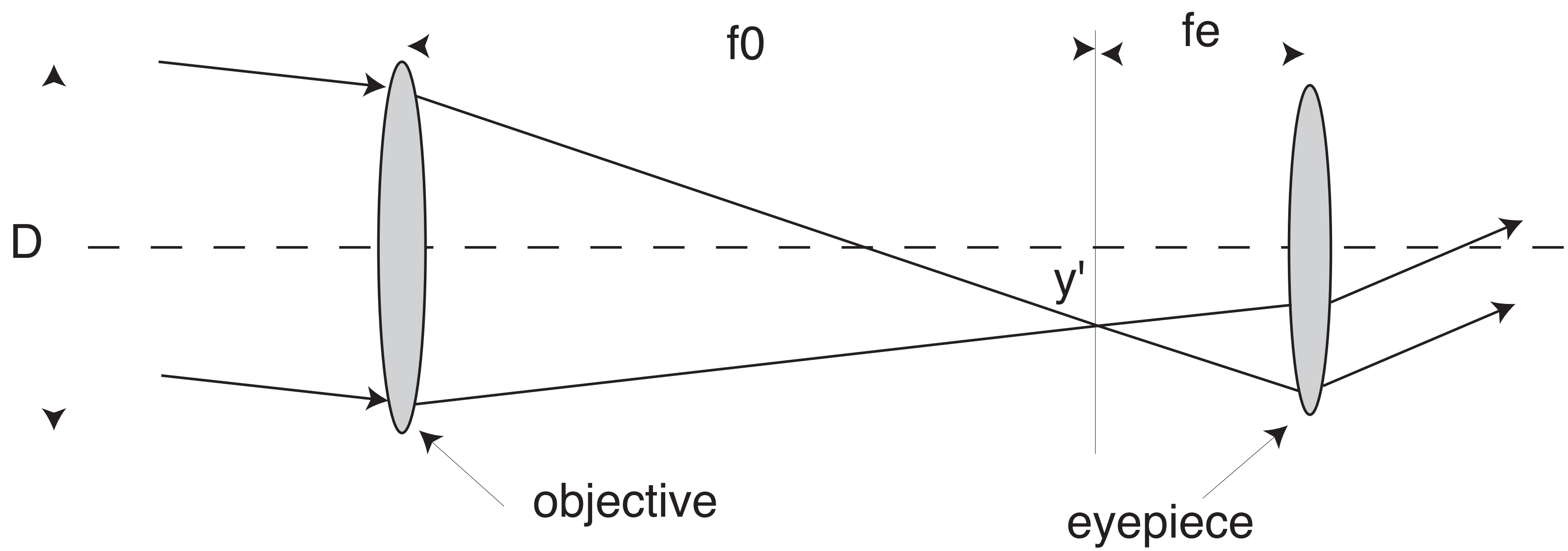


Lateral resolution = y'

$$\text{Angular resolution} = \frac{y'}{f0}$$

$$= 0.61 \frac{\lambda}{D/2} \quad \text{(Rayleigh resolution)}$$

Telescope: Matrix Formulation



$$\text{system matrix} = \left[\begin{matrix}\text{thin lens}\\ \text{(eyepiece)}\end{matrix}\right]\left[\begin{matrix}\text{propagation}\\ \text{through d}\end{matrix}\right]\left[\begin{matrix}\text{thin lens}\\ \text{(objective)}\end{matrix}\right]$$

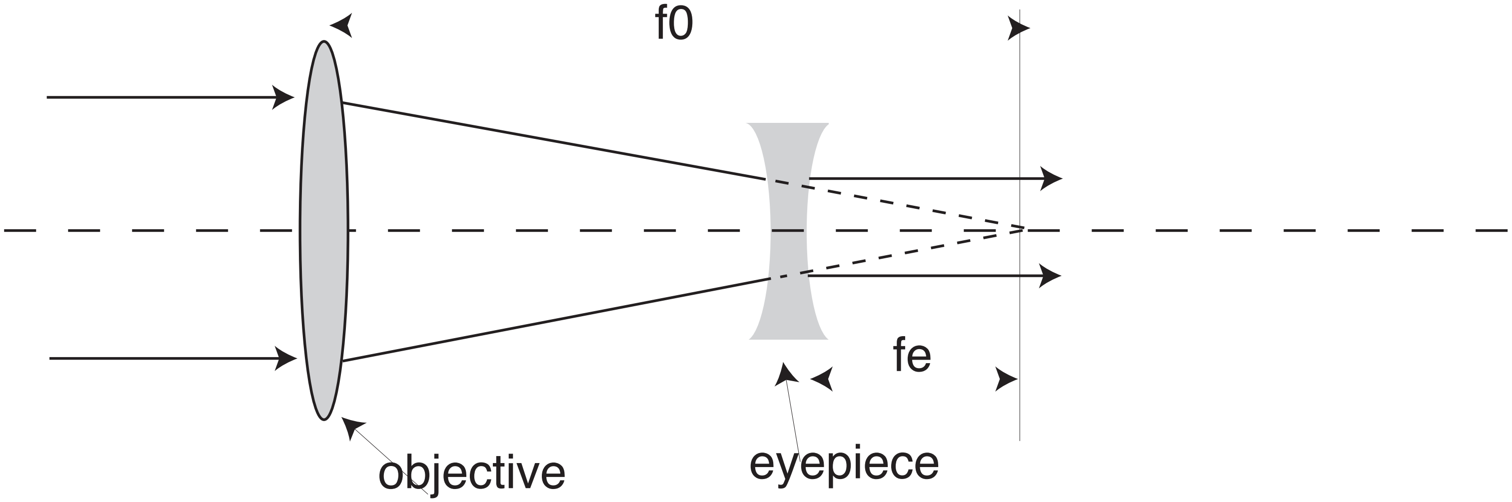
power = 0

$$= \begin{pmatrix} 1 & -\frac{1}{fe} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ d & 1 \end{pmatrix}\begin{pmatrix} 1 & -\frac{1}{f0} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1-\frac{d}{fe} & \frac{d}{f0\,fe}-\frac{1}{fe}-\frac{1}{f0} \\ d & 1-\frac{d}{f0} \end{pmatrix} = \begin{pmatrix} -\frac{f0}{fe} & 0 \\ d & -\frac{fe}{f0} \end{pmatrix}$$

d = f0+fe

angular
magnification

Galilean Telescope



Newtonian Telescope