Modern View of Light

- Photon = elementary particle
  mass = 0
  speed $c = 3 \times 10^8$ m/s

- Wave-particle duality
  Energy $E = h\nu$
  Momentum $p = \frac{h}{\lambda}$

$h = \text{Planck's constant}$

- $E = h\nu$

$h = 6.626 \times 10^{-34}$ J s

- Dispersion relation
  $c = \lambda\nu$

$\lambda = \text{wavelength (m)}$

$\nu = \text{frequency (1/s)}$

Temporal oscillation
spatial period of the light waves
The concept of a "ray"

Wavefronts
(planes of constant phase)

\( t = 0 \)
The concept of a "ray"

In homogeneous media, light propagates in rectilinear paths.
Light in matter

Speed
\[ c = 3 \times 10^8 \text{ m/s} \]

\[ v = \frac{c}{n} \]

\( n \) = index of refraction
(or refractive index)

Absorption Coefficient
\[ \alpha = 0 \]

energy transmitted through
length \( L = \exp(-2\alpha L) \)

Example: Glass has \( n \sim 1.5 \), glass fiber has \( \alpha \sim 0.0288 \text{ /km} \)
Reflection

- $\theta_i = \theta_r$
- Normal, incident & reflected rays lie in one plane
Refraction

- $n_1 \sin \theta_i = n_2 \sin \theta_t$
- Snell's Law
- Incident, reflected & refracted rays lie in one plane
Total Internal Reflection

- $n_1 > n_2$
- when $\theta_i = \theta_c = \sin^{-1} \frac{n_2}{n_1}$
  $\theta_t = 90^0$
- when $\theta_i > \theta_c$
  all light is reflected
Frustrated Total Internal Reflection (FTIR)

- Medium 1
- Medium 2
- Medium 3

- Incident light
- Reflected light
- Normal

\[ \theta_i > \theta_c \]

- \( n_1 > n_2 \)
- Refracted beam is evanescent
- Light "tunnels" into 3rd medium

\[ n_3 = n_1 \]
Geometrical Optics

- $\lambda$, wavelength is small
- Wave effects such as interference & diffraction ignored
- Simple analysis, yet sufficient for many situations

. . . as opposed to

Physical Optics

- $\lambda$ is non-zero
Huygen's Principle

point source

primary wavefront
Huygen's Principle

point source

primary wavefront

secondary point source
Huygen's Principle

point source
primary wavefront
secondary point source
secondary wavefront
Huygen's Principle

- Each point on a wavefront acts as a secondary point source emanating spherical wavelets.
- The wavefront after a short propagation distance is the superposition of all the spherical wavelets.
Why Imaging Systems are Necessary?

- Each point on an object scatters incident illumination into a spherical wavelet according to Huygen's Principle.
- At a short distance from the object, the wavelets from all the points get entangled and object details are delocalized.
- The objective of imaging is to relocalize the object details by assigning ("focusing") rays from a single object point to a single "image" point.
Lens: Main Instrument for Image Formation

- The curved surface makes the rays bend at angles proportional to their distance from the optical axis, according to Snell's law. Thus a diverging wavefront becomes converging on the output side.
Analyzing Lenses: Ray Tracing

point source (object)

free space propagation in air

refraction at air-glass interface

glass

free space propagation in glass

refraction at glass-air interface

air

free space propagation in air

point image

optical axis
Paraxial Approximation

Only rays close to the optical axis are considered

$\varepsilon \ll 1 \text{ rad}$

1st order Taylor approximations apply

$\sin \varepsilon \approx \varepsilon \quad \tan \varepsilon \approx \varepsilon \quad \cos \varepsilon \approx 1$

Valid for $\varepsilon$ upto 10-30 degrees
Paraxial Approximation

- Apply Snell's law assuming refraction occurred at the intersection of the optical axis and the lens
- Ignore the distance between the actual off-axis ray intersection & the optical-axis intersection with the lens

Valid for small curvatures & thin optical elements
Example: 1 Spherical Surface

point source (object)

medium 1
index = n

e
optical axis
center of spherical surface

medium 2
index = n'

R = radius of curvature

free-space propagation
refraction
free-space propagation

off-axis ray
paraxial approx.
gives large error
Example: 1 Spherical Surface

- $x_0$, $x_1$, $x_2$ are points on the surface.
- $\alpha_0 = \alpha_1$ is the angle of incidence.
- $\alpha_2$ is the angle of refraction.
- $D_{01}$ and $D_{12}$ are distances.
- $R$ is the radius of curvature.
- $n$ and $n'$ are the indices of refraction.

$R = \text{radius of curvature}$
Example: 1 Spherical Surface

Starting Location: Position $x_0$  Direction $\alpha_0$

Propagation through distance $D_{01}$ \[ \begin{cases} x_1 = x_0 + D_{01} \alpha_0 \\ \alpha_1 = \alpha_0 \end{cases} \]

Refraction at spherical interface \[ \begin{cases} x'_1 = x_1 \\ \alpha'_1 = \frac{n}{n'} \alpha_1 + \frac{n - n'}{n'R} x_1 \end{cases} \]

$R =$ radius of curvature
Example: 1 Spherical Surface

\[ \alpha_0 = \alpha_1 \]

\[ \alpha'_1 = \alpha_2 \]

\[ R = \text{radius of curvature} \]

Propagation through distance \( D_{12} \)

\[ \begin{align*}
    x_2 &= x_1 + D_{12} \alpha'_1 \\
    \alpha_2 &= \alpha'_1
\end{align*} \]

Putting together . . .
Example: 1 Spherical Surface

\[ x_2 = \left( \frac{n - n'}{n'} \right) \frac{D_{12}}{R} + 1 \right) x_0 + \left( D_{01} + \frac{nD_{12}}{n'} + \frac{n - n'}{n'} \frac{D_{01}D_{12}}{R} \right) \alpha_0 \]

\[ \alpha_2 = \left( \frac{n - n'}{n'R} \right) x_0 + \left( \frac{n}{n'} + \frac{n - n'}{n'} \frac{D_{10}}{R} \right) \alpha_0 \]
Sign Conventions

- Light travels from left to right
- Radius of curvature is positive when surface is convex towards left
- Longitudinal distances are positive if pointing to the right
- Lateral distances are positive when pointing up
- Ray angles are positive if ray direction is obtained by rotating the optical axis (+z) counter-clockwise through an acute angle
On-axis Image Formation

All rays emanating from S converge to P irrespective of angle, \( \alpha_0 \).

\[
\frac{n'}{D_{12}} + \frac{n}{D_{01}} = \frac{n' - n}{R}
\]

"power" of spherical surface (units= Diopters; 1D = 1/m)
Image of Point Object at Infinity

\[ D_{12} = f' = \frac{n' R}{n' - n} \]

Image Focal Length
Point Object Imaged at Infinity

\[ D_{01} = f = \frac{n R}{n' - n} \]

Object Focal Length

\((x_0 = 0)\)

\(S\)
Matrix Formulation

\[
\begin{bmatrix}
  n_{\text{out}} & \alpha_{\text{out}} \\
  x_{\text{out}} & \n \\
\end{bmatrix}
= 
\begin{bmatrix}
  M_{11} & M_{12} \\
  M_{21} & M_{22} \\
\end{bmatrix}
\begin{bmatrix}
  n_{\text{in}} & \alpha_{\text{in}} \\
  x_{\text{in}} & \n \\
\end{bmatrix}
\]

Translation through Uniform Medium

\[
\begin{bmatrix}
  n & \alpha_1 \\
  x_1 & \n \\
\end{bmatrix}
= 
\begin{bmatrix}
  1 & 0 \\
  \frac{D_{01}}{n} & 1 \\
\end{bmatrix}
\begin{bmatrix}
  n & \alpha_0 \\
  x_0 & \n \\
\end{bmatrix}
\]

Refraction by Spherical Surface

\[
\begin{bmatrix}
  n' & \alpha'_1 \\
  x'_1 & \n \\
\end{bmatrix}
= 
\begin{bmatrix}
  1 & -\left(\frac{n' - n}{R}\right) \\
  0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
  n & \alpha_1 \\
  x_1 & \n \\
\end{bmatrix}
\]

power
Example: 1 Spherical Surface

\[ \begin{bmatrix} n' \alpha'_2 \\ x'_2 \end{bmatrix} = \begin{bmatrix} \text{Translation through } D_{01} \\ \text{Refraction at spherical interface} \end{bmatrix} \times \begin{bmatrix} \text{Translation through } D_{12} \end{bmatrix} \times \begin{bmatrix} n \alpha_0 \\ x_0 \end{bmatrix} \]

\[ x_2 = \left( \frac{n - n'}{n'} \frac{D_{12}}{R} + 1 \right) x_0 + \left( D_{01} + \frac{nD_{12}}{n'} + \frac{n - n'}{n'} \frac{D_{01}D_{12}}{R} \right) \alpha_0 \]

\[ \alpha_2 = \left( \frac{n - n'}{n'R} \right) x_0 + \left( \frac{n}{n'} + \frac{n - n'}{n'} \frac{D_{10}}{R} \right) \alpha_0 \]
Thin Lens

\[
\begin{align*}
\begin{bmatrix}
\alpha'_{\text{out}} \\
X'_{\text{out}}
\end{bmatrix} &= \text{Refraction at 1st spherical interface} \\
& \times \text{Refraction at 2nd spherical interface} \\
& \times \begin{bmatrix}
\alpha_{\text{in}} \\
X_{\text{in}}
\end{bmatrix}
\end{align*}
\]
Thin Lens

\[
\begin{pmatrix}
\alpha'_{\text{out}} \\
\chi'_{\text{out}}
\end{pmatrix} = \begin{pmatrix}
1 & -\left(\frac{1 - n'}{R'}\right) \\
0 & 1
\end{pmatrix} \times \begin{pmatrix}
1 & -\left(\frac{n' - 1}{R}\right) \\
0 & 1
\end{pmatrix} \times \begin{pmatrix}
\alpha_{\text{in}} \\
\chi_{\text{in}}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
1 & -\left(\frac{n' - 1}{R} + \frac{1 - n'}{R'}\right) \\
0 & 1
\end{pmatrix}
\]

\[P_{\text{thin-lens}} = (n' - 1)\left(\frac{1}{R} - \frac{1}{R'}\right)\]

Lens-maker's formula
Power of Surfaces

- Positive power bends rays "inwards"
  - Plano-convex lens ($R > 0$)
  - Bi-convex Lens ($R > 0, R < 0$)

- Negative power bends rays "outwards"
  - Plano-concave lens ($R < 0$)
  - Bi-concave Lens ($R < 0, R > 0$)
Power of Surfaces

- Matrix Formulation

\[ \text{Power} = - M_{12} \]

- Power & Focal Length

\[ f = 1 / \text{power} \]
Thick Lens: Principal Planes

Note: In paraxial approximation, principal and focal planes are flat, whereas in reality these are curved surfaces (not spherical).
Thick Lens: Focal Lengths

1st focal plane
1st principal plane
2nd principal plane
2nd focal plane
optical axis

FFL = Front Focal Length
BFL = Back Focal Length
EFL = Effective Focal Length

EFL = Effective Focal Length
Significance of Principal Planes

2nd principal plane

complex optical system

optical axis

2nd focus

2nd principal plane

optical axis
Significance of Principal Planes

1st focus

1st principal plane

complex optical system

optical axis
Thick Lens: Matrix Transformation

\[
\begin{bmatrix}
\alpha_{\text{out}}' \\
x_{\text{out}}'
\end{bmatrix}
= \begin{bmatrix}
1 & -\left(\frac{1 - n'}{R_2}\right) \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
\frac{D_l}{n'} & 1
\end{bmatrix}
\begin{bmatrix}
1 & -\left(\frac{n' - 1}{R_1}\right) \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\alpha_{\text{in}} \\
x_{\text{in}}
\end{bmatrix}
\]
Thick Lens: Matrix Transformation

optical axis

Dl
refraction
(radius of curvature = R1)

propagation through Dl

refraction
(radius of curvature = R2)

\[
\begin{bmatrix}
\alpha'_{\text{out}} \\
\chi'_{\text{out}}
\end{bmatrix} =
\begin{bmatrix}
1 + \frac{D_l}{n'} \left( \frac{n' - 1}{R_2} \right) & - (n' - 1) \left\{ \frac{1}{R_1} - \frac{1}{R_2} + (n' - 1) \frac{D_l}{n'R_1R_2} \right\} \\
\frac{D_l}{n'} & 1 - \frac{D_l}{n'} \left( \frac{n' - 1}{R_1} \right)
\end{bmatrix}
\begin{bmatrix}
\alpha_{\text{in}} \\
\chi_{\text{in}}
\end{bmatrix}
\]
Thick Lens: Matrix Transformation

optical axis

refraction (radius of curvature = $R_1$)

propagation through $D_l$

refraction (radius of curvature = $R_2$)

$$EFL = f \quad \frac{1}{f} = (n' - 1)\left\{ \frac{1}{R_1} - \frac{1}{R_2} + (n' - 1)\frac{D_l}{n'R_1R_2} \right\}$$
Image point is located at the common intersection of all rays emanating from the corresponding object point.

- Rays passing through the two focal points (focii), and the chief ray can be ray-traced directly.
Imaging Condition: Matrix Form

\[
\begin{bmatrix}
1 & 0 \\
S'/n' & 1 \\
\end{bmatrix}
\begin{bmatrix}
1 & -P \\
0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
S/n & 1 \\
\end{bmatrix}
= 
\begin{bmatrix}
1 - PS/n & -P \\
S'/n' + S/n - PSS'/nn' & 1 - PS'/n' \\
\end{bmatrix}
\]
Imaging Condition: Matrix Form

1st focus

2nd focus

object

image

1st principal plane

2nd principal plane

optical axis

\[
\begin{bmatrix}
 n & \alpha' \\
 x' \\
\end{bmatrix} =
\begin{bmatrix}
 1 - PS/n & -P \\
 S'/n' + S/n - PSS'/nn' & 1 - PS'/n' \\
\end{bmatrix}
\begin{bmatrix}
 n & \alpha \\
 x \\
\end{bmatrix}
\]

x' must be independent of \( \alpha \)
Imaging Condition: Matrix Form

\[ \frac{n}{S} + \frac{n'}{S'} = P \]

System immersed in air
\[ \frac{1}{S} + \frac{1}{S'} = \frac{1}{f} \]
\[ f = \text{EFL} \]
Lateral Magnification

When Imaging Condition is satisfied,

\[ m_x = \frac{x'}{x} = 1 - \frac{PS'}{n'} \]
Angular Magnification

When Imaging Condition is satisfied,

\[ m_a = \frac{\Delta \alpha'}{\Delta \alpha} = \frac{n}{n'}(1 - \frac{PS'}{n'}) \]
**Generalized Imaging Conditions**

\[
\begin{pmatrix}
n \alpha' \\
x'
\end{pmatrix}
= \begin{pmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{pmatrix}
\begin{pmatrix}
n \alpha \\
x
\end{pmatrix}
\]

- **Image**: \(n \alpha'\) and \(x'\)
- **System Matrix**: \(M_{ij}\)
- **Object**: \(n \alpha\) and \(x\)

**Power**

\[P = -M_{12}\]

**Imaging Condition**

\[M_{21} = 0\]

**Lateral Magnification**

\[m_x = M_{22}\]

**Angular Magnification**

\[m_a = \frac{n}{n'} M_{11}\]
Aperture Stop & Field Stop

- AS limits the amount of light energy reaching each image point
- FS limits the number of image points (extent of the image)
Entrance Pupil is the image of the AS seen from the object side.
Exit Pupil is the image of the AS seen from the image side.
Mirrors & prisms

• Last time: optical elements,
  – Lenses
    • Basic properties of spherical surfaces
    • Ray tracing
    • Image formation
    • Magnification
  – Mirrors

• Today: more optical elements,
  – Prisms
  – Mirrors
Lens: main instrument for image formation

The curved surface makes the rays bend proportionally to their distance from the “optical axis”, according to Snell’s law. Therefore, the divergent wavefront becomes convergent at the right-hand (output) side.
Cardinal Planes and Points

- Rays generated from axial point at infinity (*i.e.*, forming a ray bundle parallel to the optical axis) and entering an optical system intersect the optical axis at the **Focal Points**.
- The intersection of the extended entering parallel rays and the extended exiting convergent rays forms the **Principal Surface** (**Plane** in the paraxial approximation.)
- The extension of a ray which enters and exits the optical system with the same angle of propagation intersects the optical axis at the **Nodal Points**.

![Diagram of cardinal planes and points]
Recap of lens-like instruments

- **Cardinal Points and Focal Lengths**

  \[
  \begin{pmatrix}
  n' \\
  x'
  \end{pmatrix} =
  \begin{pmatrix}
  M_{11} & M_{12} \\
  M_{21} & M_{22}
  \end{pmatrix}
  \begin{pmatrix}
  n \\
  x
  \end{pmatrix}
  \]

  **Matrix formulation**

- **Imaging conditions**

  \[M_{12} \neq 0\]

  \[P = -M_{12} \neq 0\]

  \[M_{21} = 0\]

  **Magnification**

  lateral \[m_x = M_{22}\]

  angular \[m_a = \frac{n}{n'} M_{11}\]
Prisms

Air → Glass → Air

Total Internal Reflection (TIR)

Air → Glass → Air
Assume a symmetric case,

\[ a = b \]
\[ a' = b' \]

\[ a' = \frac{\theta}{2} \]
\[ a = \frac{\theta + D}{2} \]

From Snell's law,

Prism Equation

\[ n = \frac{\sin\left(\frac{D + \theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)} \]
Dispersion

Refractive index $n$ is function of the wavelength

Newton’s prism

white light (all visible wavelengths)

glass

air

red
green
blue

Newton’s prism
Dispersion measures

Reference color lines
C (H- $\lambda$=656.3nm, red), D (Na- $\lambda$=589.2nm, yellow),
F (H- $\lambda$=486.1nm, blue)

Crown glass has

$n_f = 1.52933$  \hspace{1cm} $n_D = 1.52300$  \hspace{1cm} $n_C = 1.52042$

Dispersive power \hspace{1cm} $V = \frac{n_F - n_C}{n_D - 1}$

Dispersive index \hspace{1cm} $\nu = \frac{1}{V} = \frac{n_D - 1}{n_F - n_C}$
Mirrors: the law of reflection

\[ \theta \]
Plane Mirrors

Plane Mirrors have zero power.

Images are laterally inverted.

Image is always virtual.
Sign conventions for reflection

• Light travels from left to right before reflection and from right to left after reflection
• A radius of curvature is positive if the surface is convex towards the left
• Longitudinal distances before reflection are positive if pointing to the right; longitudinal distances after reflection are positive if pointing to the left
• Longitudinal distances are positive if pointing up
• Ray angles are positive if the ray direction is obtained by rotating the +z axis counterclockwise through an acute angle
Example: spherical mirror

In the paraxial approximation, it (approximately) focuses an incoming parallel ray bundle (from infinity) to a point.
Reflective optics formulae

Imaging condition
\[ \frac{1}{D_{12}} + \frac{1}{D_{01}} = -\frac{2}{R} \]

Focal length
\[ f = -\frac{R}{2} \]

Magnification
\[ m_x = -\frac{D_{12}}{D_{01}} \quad m_\alpha = -\frac{D_{01}}{D_{12}} \]
Paraboloid mirror: perfect focusing
(e.g. satellite dish)

What should the shape function $s(x)$ be in order for the incoming parallel ray bundle to come to perfect focus?
Aberrations

- Deviation of the wavefront from its ideal spherical shape due to the imperfect refraction/reflection by the optical elements.

- Optical elements (lenses, mirrors) produce perfect spherical wavefronts only in the paraxial approximation (i.e. for small angles of propagation with respect to the optical axis).

- At larger angles, Seidel (or primary) aberrations occur.
Chromatic Aberration

- Index of refraction varies with wavelength (index for blue > index for red)

- Blue light comes to focus closer to the lens than red light. The horizontal distance between the two images is called *longitudinal chromatic aberration*.

Images formed by different wavelengths also have different transverse (lateral) magnifications. This is called *lateral chromatic aberration*.

- Image formed by blue light is smaller and closer to the lens than that formed by red light.
Correcting Chromatic Aberration

Using a combination of two kinds of glasses, crown and flint to form a dichromat. Crown can have more +ve power and has moderate dispersion, while flint can have lower -ve power and has high dispersion.

\[
P = P_1 + P_2
\]

\[
P = (n_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)
\]

Set the powers equal for the wavelengths, F, C and e.

\[
\frac{P_1}{P_2} = \frac{V_1}{V_2}
\]

\[
V = \frac{n_e - 1}{n_F - n_C} \text{ (dispersion factor or Abbe's factor)}
\]

\[
P_1 = P \frac{V_1}{V_1 - V_2} \quad P_2 = -P \frac{V_2}{V_1 - V_2}
\]
## Summary of Aberrations

<table>
<thead>
<tr>
<th>Aberration</th>
<th>Character</th>
<th>Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical aberration</td>
<td>monochromatic, on- and off-axis, image blur</td>
<td>Aspherics, doublet, high-index</td>
</tr>
<tr>
<td>Coma</td>
<td>monochromatic, off-axis only, blur</td>
<td>spaced doublet with central stop</td>
</tr>
<tr>
<td>Oblique astigmatism</td>
<td>monochromatic, off-axis, blur</td>
<td>spaced doublet with stop</td>
</tr>
<tr>
<td>Curvature of field</td>
<td>monochromatic, off-axis</td>
<td>spaced doublet</td>
</tr>
<tr>
<td>Distortion</td>
<td>monochromatic, off-axis</td>
<td>spaced doublet with stop</td>
</tr>
<tr>
<td>Chromatic aberration</td>
<td>polychromatic, on- and off-axis, blur</td>
<td>contact doublet, spaced doublet</td>
</tr>
</tbody>
</table>
Optical Systems

General criterion while designing an optical system:

- Light gathering power (capacity to form a bright image)
- Magnification
- Resolving power (capacity to form sharp images of small detail)
- Others such as physical size, weight, cost etc.

Astronomical Telescope

- Light from distant star
- Objective
- Eyepiece
Angular Magnification of Telescope

Light from distant star

Objective

Eyepiece

\[ M = \frac{\text{angular size of image}}{\text{angular size of object}} = \frac{\tan \theta'}{\tan \theta} \]

\[ = \frac{y'/-fe}{y'/f0} = -\frac{f0}{fe} \]
Angular Magnification of Telescope

\[ M = \frac{\text{diameter of entrance pupil}}{f_0} = \frac{\text{diameter of entrance pupil}}{-f_e} \]

Objective

Eyepiece

Entrance pupil

Exit pupil
Angular Resolution of Telescope

Lateral resolution = $y'$

Angular resolution = \[ \frac{y'}{f_0} \]

= $0.61 \frac{\lambda}{D/2}$ (Rayleigh resolution)
Telescope: Matrix Formulation

don't hallucinate.

System matrix = \[
\begin{bmatrix}
\text{thin lens (eyepiece)} & \text{propagation through } d & \text{thin lens (objective)}
\end{bmatrix}
\]

\[
= \begin{pmatrix}
1 & -\frac{1}{fe} \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
d & 1
\end{pmatrix}
\begin{pmatrix}
1 & -\frac{1}{f0} \\
0 & 1
\end{pmatrix}
= \begin{pmatrix}
1-\frac{d}{fe} & \frac{d}{f0} - \frac{1}{fe} - \frac{1}{f0} \\
d & 1-\frac{d}{f0}
\end{pmatrix}
= \begin{pmatrix}
-\frac{f0}{fe} & 0 \\
f0 & -\frac{fe}{f0}
\end{pmatrix}
\]

\(d = f0 + fe\)

angular magnification

power = 0