Modern View of Light

- Photon = elementary particle
  - mass = 0
  - speed \( c = 3 \times 10^8 \) m/s
- Wave-particle duality
  - Energy \( E = h\nu \)
  - Momentum \( p = \frac{h}{\lambda} \)
- Planck's constant \( h = 6.626 \times 10^{-34} \) J s
- Dispersion relation \( c = \lambda\nu \)
  - \( \lambda \) = wavelength (m)
  - Spatial period of the light waves
- Frequency \( \nu \) = frequency (1/s)
  - Temporal oscillation
  - Frequency of light waves
The concept of a "ray"
The concept of a "ray"

In homogeneous media, light propagates in rectilinear paths.
Light in matter

vacuum

matter

Speed

\[ c = 3 \times 10^8 \text{ m/s} \]

\[ v = \frac{c}{n} \]

\[ n = \text{index of refraction} \]

(or refractive index)

Absorption Coefficient

\[ \alpha = 0 \]

energy transmitted through length \( L = \exp(-2\alpha L) \)

Example: Glass has \( n \sim 1.5 \), glass fiber has \( \alpha \sim 0.0288 \text{ /km} \)
Reflection

medium 1

incident light

normal

reflected light

θᵢ  θᵣ

θᵢ = θᵣ

Normal, incident & reflected rays lie in one plane
Refraction

- Incident, reflected & refracted rays lie in one plane

Snell's Law

\[ n_1 \sin \theta_i = n_2 \sin \theta_t \]
Total Internal Reflection

medium 1
\( n_1 \)

medium 2
\( n_2 \)

incident light

normal

reflected light

\( \theta_i \)

\( \theta_t \)

\( \theta_c \)

\( n_1 > n_2 \)

\( \text{when } \theta_i = \theta_c = \sin^{-1} \frac{n_2}{n_1} \)

\( \theta_t = 90^\circ \)

\( \text{when } \theta_i > \theta_c \)

all light is reflected
Frustrated Total Internal Reflection (FTIR)

- medium 1
- medium 2
- medium 3

incident light

reflected light

\( n_1 \) > \( n_2 \)

\( \theta_i > \theta_c \)

refracted beam is evanescent

Light "tunnels" into 3rd medium
Geometrical Optics

- $\lambda$, wavelength is small
- Wave effects such as interference & diffraction ignored
- Simple analysis, yet sufficient for many situations

... as opposed to

Physical Optics

- $\lambda$ is non-zero
Huygen's Principle

point source

primary wavefront
Huygen's Principle
Huygen's Principle

primary wavefront

secondary point source

secondary wavefront

point source
Huygen's Principle

- Each point on a wavefront acts as a secondary point source emanating spherical wavelets.
- The wavefront after a short propagation distance is the superposition of all the spherical wavelets.
Why Imaging Systems are Necessary?

- Each point on an object scatters incident illumination into a spherical wavelet according to Huygen's Principle.
- At a short distance from the object, the wavelets from all the points get entangled and object details are delocalized.
- The objective of imaging is to relocalize the object details by assigning ("focusing") rays from a single object point to a single "image" point.
The curved surface makes the rays bend at angles proportional to their distance from the optical axis, according to Snell's law. Thus a diverging wavefront becomes converging on the output side.
Analyzing Lenses: Ray Tracing

point source (object) → air → glass → air → point image

- free space propagation in air
- refraction at air-glass interface
- refraction at glass-air interface
- free space propagation in glass
- free space propagation in air

optical axis
Paraxial Approximation

- Only rays close to the optical axis are considered
  \[ \varepsilon \ll 1 \text{ rad} \]
- 1st order Taylor approximations apply
  \[ \sin \varepsilon \approx \varepsilon \quad \tan \varepsilon \approx \varepsilon \quad \cos \varepsilon \approx 1 \]
- Valid for \( \varepsilon \) upto 10-30 degrees
- Apply Snell's law assuming refraction occurred at the intersection of the optical axis and the lens.
- Ignore the distance between the actual off-axis ray intersection & the optical-axis intersection with the lens.

Valid for small curvatures & thin optical elements
Example: 1 Spherical Surface

- Point source (object)
- Medium 1, index = n
- Medium 2, index = n'
- Optical axis
- Center of spherical surface
- R = radius of curvature
- Free-space propagation
- Refraction
- Off-axis ray, paraxial approx. gives large error
Example: 1 Spherical Surface

- \( n \) and \( n' \): optical axis
- \( R \): radius of curvature
- \( D_{01} \) and \( D_{12} \)
- \( x_0, x_1, x_2 \)
- \( \alpha_0 = \alpha_1 \)
- \( \alpha_2 \)

R = radius of curvature
Example: 1 Spherical Surface

Starting Location: Position $x_0$ Direction $\alpha_0$

Propagation through distance $D_{01}$

$$\left\{ \begin{align*}
x_1 &= x_0 + D_{01} \alpha_0 \\
\alpha_1 &= \alpha_0
\end{align*} \right.$$

Refraction at spherical interface

$$\left\{ \begin{align*}
x'_1 &= x_1 \\
\alpha'_1 &= \frac{n}{n'} \alpha_1 + \frac{n - n'}{n'R} x_1
\end{align*} \right.$$

$R =$ radius of curvature
Example: 1 Spherical Surface

\[ x_2 = x_1 + D_{12} \alpha'_1 \]
\[ \alpha_2 = \alpha'_1 \]

Propagation through distance \( D_{12} \)

Putting together . . .
Example: 1 Spherical Surface

\[ x_2 = \left( \frac{n - n'}{n'} \right) \frac{D_{12}}{R} + 1 \right) x_0 + \left( D_{01} + \frac{nD_{12}}{n'} + \frac{n - n'}{n'} \frac{D_{01}D_{12}}{R} \right) \alpha_0 \]

\[ \alpha_2 = \left( \frac{n - n'}{n'R} \right) x_0 + \left( \frac{n}{n'} + \frac{n - n'}{n'} \frac{D_{10}}{R} \right) \alpha_0 \]
Sign Conventions

- Light travels from left to right
- Radius of curvature is positive when surface is convex towards left
- Longitudinal distances are positive if pointing to the right
- Lateral distances are positive when pointing up
- Ray angles are positive if ray direction is obtained by rotating the optical axis (+z) counter-clockwise through an acute angle
On-axis Image Formation

All rays emanating from S converge to P irrespective of angle, $\alpha_0$

$$\frac{n'}{D_{12}} + \frac{n}{D_{01}} = \frac{n' - n}{R}$$

"power" of spherical surface (units = Diopters; 1D = 1/m)
Image of Point Object at Infinity

\[ D_{12} = f' = \frac{n'R}{n' - n} \]

Image Focal Length
Point Object Imaged at Infinity

\[ D_{01} = f = \frac{n \cdot R}{n' - n} \]

Object Focal Length
Matrix Formulation

\[
\begin{bmatrix}
  n_{\text{out}} & \alpha_{\text{out}} \\
  x_{\text{out}}
\end{bmatrix} =
\begin{bmatrix}
  M_{11} & M_{12} \\
  M_{21} & M_{22}
\end{bmatrix}
\begin{bmatrix}
  n_{\text{in}} & \alpha_{\text{in}} \\
  x_{\text{in}}
\end{bmatrix}
\]

Translation through Uniform Medium

\[
\begin{bmatrix}
  n & \alpha_1 \\
  x_1
\end{bmatrix} =
\begin{bmatrix}
  1 & 0 \\
  \frac{D_{01}}{n} & 1
\end{bmatrix}
\begin{bmatrix}
  n & \alpha_0 \\
  x_0
\end{bmatrix}
\]

Refraction by Spherical Surface

\[
\begin{bmatrix}
  n' & \alpha'_1 \\
  x'_1
\end{bmatrix} =
\begin{bmatrix}
  1 & -\left(\frac{n' - n}{R}\right) \\
  0 & 1
\end{bmatrix}
\begin{bmatrix}
  n & \alpha_1 \\
  x_1
\end{bmatrix}
\]
Example: 1 Spherical Surface

\[ x_2 = \left( \frac{n - n'}{n'} \frac{D_{12}}{R} + 1 \right) x_0 + \left( D_{01} + \frac{n D_{12}}{n'} + \frac{n - n'}{n'} \frac{D_{01} D_{12}}{R} \right) \alpha_0 \]

\[ \alpha_2 = \left( \frac{n - n'}{n'R} \right) x_0 + \left( \frac{n}{n'} + \frac{n - n'}{n'R} \frac{D_{10}}{R} \right) \alpha_0 \]
Thin Lens

\[ \frac{\alpha'_{\text{out}}}{\alpha_{\text{in}}} = \text{Refraction at 2nd spherical interface} \times \text{Refraction at 1st spherical interface} \times \frac{\alpha_{\text{in}}}{\alpha_{\text{in}}} \]

\[ \frac{x'_{\text{out}}}{x_{\text{in}}} = \text{optical axis} \]

\[ R = \text{radius of curvature} \]
Thin Lens

\[
\begin{align*}
\begin{bmatrix}
\alpha'_{\text{out}} \\
\chi'_{\text{out}}
\end{bmatrix}
&= 
\begin{bmatrix}
1 & -\left(\frac{1 - n'}{R'}\right) \\
0 & 1
\end{bmatrix}
\times
\begin{bmatrix}
1 & -\left(\frac{1}{R}\right) \\
0 & 1
\end{bmatrix}
\times
\begin{bmatrix}
\alpha_{\text{in}} \\
\chi_{\text{in}}
\end{bmatrix} \\
&= 
\begin{bmatrix}
1 & -\left(\frac{n' - 1}{R} + \frac{1 - n'}{R'}\right) \\
0 & 1
\end{bmatrix}
\end{align*}
\]

\[P_{\text{thin-lens}} = (n' - 1)\left(\frac{1}{R} - \frac{1}{R'}\right)\]

Lens-maker's formula
Power of Surfaces

- Positive power bends rays "inwards"
  - Plano-convex lens: $R > 0$
  - Bi-convex Lens: $R > 0$, $R < 0$

- Negative power bends rays "outwards"
  - Plano-concave lens: $R < 0$
  - Bi-concave Lens: $R < 0$, $R > 0$
Power of Surfaces

- Matrix Formulation

\[
\text{Power} = - M_{12}
\]

- Power & Focal Length

\[
f = 1 / \text{power}
\]
Thick Lens: Principal Planes

Note: In paraxial approximation, principal and focal planes are flat, whereas in reality these are curved surfaces (not spherical).
Thick Lens: Focal Lengths

FFL = Front Focal Length
BFL = Back Focal Length
EFL = Effective Focal Length
Significance of Principal Planes

- 2nd principal plane
- Optical axis
- Complex optical system
- 2nd focus
- 2nd principal plane
Significance of Principal Planes

1st principal plane

complex optical system

optical axis

1st focus

1st principal plane
Thick Lens: Matrix Transformation

\[
\begin{bmatrix}
\alpha'_{\text{out}} \\
\chi'_{\text{out}}
\end{bmatrix}
= \begin{bmatrix}
1 & -\left(\frac{1 - n'}{R_2}\right) \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
\frac{D_l}{n'} & 1
\end{bmatrix}
\begin{bmatrix}
1 & -\left(\frac{n' - 1}{R_1}\right) \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\alpha_{\text{in}} \\
\chi_{\text{in}}
\end{bmatrix}
\]

refraction (radius of curvature = \(R_1\))
propagation through \(D_l\)
refraction (radius of curvature = \(R_2\))
Thick Lens: Matrix Transformation

\[ \begin{pmatrix} \alpha'_{\text{out}} \\ x'_{\text{out}} \end{pmatrix} = \begin{pmatrix} 1 + \frac{D_l}{n'} \left( \frac{n' - 1}{R_2} \right) - (n' - 1) \left\{ \frac{1}{R_1} - \frac{1}{R_2} + (n' - 1) \frac{D_l}{n'R_1R_2} \right\} \\ \frac{D_l}{n'} \left( \frac{n' - 1}{R_1} \right) 1 - \frac{D_l}{n'} \left( \frac{n' - 1}{R_1} \right) \end{pmatrix} \begin{pmatrix} \alpha_{\text{in}} \\ x_{\text{in}} \end{pmatrix} \]
Thick Lens: Matrix Transformation

optical axis

refraction (radius of curvature $= R_1$)

propagation through $D_l$

refraction (radius of curvature $= R_2$)

$$EFL = f$$

$$\frac{1}{f} = (n' - 1)\left\{ \frac{1}{R_1} - \frac{1}{R_2} + (n' - 1)\frac{D_l}{n'R_1R_2} \right\}$$
Image point is located at the common intersection of all rays emanating from the corresponding object point.

- Rays passing through the two focal points (focii), and the *chief ray* can be ray-traced directly.
Imaging Condition: Matrix Form

object

1st focus

1st principal plane

n

S

1st principal plane

2nd principal plane

2nd focus

n'

optical axis

image

system matrix

\[
\begin{bmatrix}
1 & 0 \\ S'/n' & 1
\end{bmatrix}
\begin{bmatrix}
1 & -P \\ 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\ S/n & 1
\end{bmatrix}
= 
\begin{bmatrix}
1 - PS/n & -P \\ S'/n' + S/n - PSS'/nn' & 1 - PS'/n'
\end{bmatrix}
\]
Imaging Condition: Matrix Form

\[
\begin{bmatrix}
    n & \alpha' \\
    x'
\end{bmatrix} =
\begin{bmatrix}
    1 - PS/n & -P \\
    S'/n' + S/n - PSS'/nn' & 1 - PS'/n'
\end{bmatrix}
\begin{bmatrix}
    n & \alpha \\
    x
\end{bmatrix}
\]

\[x'\] must be independent of \(\alpha\)
Imaging Condition: Matrix Form

Imaging Condition
\[ \frac{n}{S} + \frac{n'}{S'} = P \]

System immersed in air
\[ \frac{1}{S} + \frac{1}{S'} = \frac{1}{f} \]
\[ f = \text{EFL} \]
When Imaging Condition is satisfied,

\[ m_x = \frac{x'}{x} = 1 - \frac{PS'}{n'} \]
Angular Magnification

When Imaging Condition is satisfied,

\[ m_a = \frac{\Delta \alpha'}{\Delta \alpha} = \frac{n}{n'}(1 - \frac{PS'}{n'}) \]
Generalized Imaging Conditions

\[
\begin{bmatrix}
  n \alpha' \\
  x'
\end{bmatrix}
= 
\begin{bmatrix}
  M_{11} & M_{12} \\
  M_{21} & M_{22}
\end{bmatrix}
\begin{bmatrix}
  n \alpha \\
  x
\end{bmatrix}
\]

image  system matrix  object

Power \( P = -M_{12} \)

Imaging Condition \( M_{21} = 0 \)

Lateral Magnification \( m_x = M_{22} \)

Angular Magnification \( m_a = (n/n')M_{11} \)
Aperture Stop & Field Stop

- AS limits the amount of light energy reaching each image point
- FS limits the number of image points (extent of the image)
Entrance Pupil is the image of the AS seen from the object side.
Exit Pupil

- Exit Pupil is the image of the AS seen from the image side.
Mirrors & prisms

• Last time: optical elements,
  – Lenses
    • Basic properties of spherical surfaces
    • Ray tracing
    • Image formation
    • Magnification
• Today: more optical elements,
  – Prisms
  – Mirrors
Lens: main instrument for image formation

The curved surface makes the rays bend proportionally to their distance from the “optical axis”, according to Snell’s law. Therefore, the divergent wavefront becomes convergent at the right-hand (output) side.
Cardinal Planes and Points

- Rays generated from axial point at infinity (i.e., forming a ray bundle parallel to the optical axis) and entering an optical system intersect the optical axis at the Focal Points.
- The intersection of the extended entering parallel rays and the extended exiting convergent rays forms the Principal Surface (Plane in the paraxial approximation.)
- The extension of a ray which enters and exits the optical system with the same angle of propagation intersects the optical axis at the Nodal Points.
Recap of lens-like instruments

- Cardinal Points and Focal Lengths

\[
\begin{bmatrix}
   x' \\
   n' x' \\
\end{bmatrix} =
\begin{bmatrix}
   M_{11} & M_{12} \\
   M_{21} & M_{22} \\
\end{bmatrix}
\begin{bmatrix}
   n x \\
   x \\
\end{bmatrix}
\]

Matrix formulation

- Imaging conditions

\[ M_{12} \neq 0 \]
\[ P = -M_{12} \neq 0 \]
\[ M_{21} = 0 \]

Magnification

- lateral
  \[ m_x = M_{22} \]

- angular
  \[ m_a = \frac{n}{n'} M_{11} \]
Prisms
Refracting Prism

Assume a symmetric case,

\[ a = b \]
\[ a' = b' \]

\[ a' = \frac{\theta}{2} \]
\[ a = \frac{\theta + D}{2} \]

From Snell's law,

\[ n = \frac{\sin\left(\frac{D+\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)} \]
Dispersion

Refractive index $n$ is function of the wavelength

white light (all visible wavelengths)

Newton’s prism
Dispersion measures

Reference color lines
C (H- $\lambda = 656.3$nm, red), D (Na- $\lambda = 589.2$nm, yellow),
F (H- $\lambda = 486.1$nm, blue)

Crown glass has

$n_F = 1.52933 \quad n_D = 1.52300 \quad n_C = 1.52042$

Dispersive power  $V = \frac{n_F - n_C}{n_D - 1}$

Dispersive index  $v = \frac{1}{V} = \frac{n_D - 1}{n_F - n_C}$
Mirrors: the law of reflection
Plane Mirrors have zero power.

Images are laterally inverted

Image is always virtual
Sign conventions for reflection

• Light travels from left to right before reflection and from right to left after reflection
• A radius of curvature is positive if the surface is convex towards the left
• Longitudinal distances before reflection are positive if pointing to the right; longitudinal distances after reflection are positive if pointing to the left
• Longitudinal distances are positive if pointing up
• Ray angles are positive if the ray direction is obtained by rotating the +z axis counterclockwise through an acute angle
Example: spherical mirror

In the paraxial approximation, it (approximately) focuses an incoming parallel ray bundle (from infinity) to a point.
Reflective optics formulae

Imaging condition
\[
\frac{1}{D_{12}} + \frac{1}{D_{01}} = -\frac{2}{R}
\]

Focal length
\[
f = -\frac{R}{2}
\]

Magnification
\[
m_x = -\frac{D_{12}}{D_{01}} \quad m_\alpha = -\frac{D_{01}}{D_{12}}
\]
Paraboloid mirror: **perfect focusing**
(e.g. satellite dish)

What should the shape function $s(x)$ be in order for the incoming parallel ray bundle to come to perfect focus?
Aberrations

- Deviation of the wavefront from its ideal spherical shape due to the imperfect refraction/reflection by the optical elements.

- Optical elements (lenses, mirrors) produce perfect spherical wavefronts only in the paraxial approximation (i.e. for small angles of propagation with respect to the optical axis).

- At larger angles, Seidel (or primary) aberrations occur.
Chromatic Aberration

- Index of refraction varies with wavelength (index for blue > index for red)

- Blue light comes to focus closer to the lens than red light. The horizontal distance between the two images is called *longitudinal chromatic aberration*.

Images formed by different wavelengths also have different transverse (lateral) magnifications. This is called *lateral chromatic aberration*.

- Image formed by blue light is smaller and closer to the lens than that formed by red light.
Correcting Chromatic Aberration

- Using a combination of two kinds of glasses, crown and flint to form a dichromat. Crown can have more +ve power and has moderate dispersion, while flint can have lower -ve power and has high dispersion.

\[ P = P_1 + P_2 \]

\[ P = (n_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \]

Set the powers equal for the wavelengths, F, C and e.

\[ \frac{P_1}{P_2} = \frac{V_1}{V_2} \]

\[ V = \frac{n_e - 1}{n_F - n_C} \] (dispersion factor or Abbe's factor)

\[ P_1 = P \frac{V_1}{V_1 - V_2} \quad P_2 = -P \frac{V_2}{V_1 - V_2} \]
## Summary of Aberrations

<table>
<thead>
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<th>Aberration</th>
<th>Character</th>
<th>Correction</th>
</tr>
</thead>
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<td>monochromatic, on- and off-axis, image blur</td>
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<td>Coma</td>
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</tr>
<tr>
<td>Chromatic aberration</td>
<td>polychromatic, on- and off-axis, blur</td>
<td>contact doublet, spaced doublet</td>
</tr>
</tbody>
</table>
Optical Systems

General criterion while designing an optical system:

- Light gathering power (capacity to form a bright image)
- Magnification
- Resolving power (capacity to form sharp images of small detail)
- Others such as physical size, weight, cost etc.

Astronomical Telescope
Angular Magnification of Telescope

M = \frac{\text{angular size of image}}{\text{angular size of object}} = \frac{\tan \theta'}{\tan \theta} = \frac{y'/-fe}{y'/f0} = -\frac{f0}{fe}
Angular Magnification of Telescope

\[ M = -\frac{f_0}{f_e} = \frac{\text{diameter of entrance pupil}}{\text{diameter of exit pupil}} \]
Angular Resolution of Telescope

Lateral resolution = y'

Angular resolution = \frac{y'}{f_0} = 0.61 \frac{\lambda}{D/2} \text{ (Rayleigh resolution)}
Telescope: Matrix Formulation

\[
\text{system matrix} = \begin{bmatrix}
\text{thin lens (eyepiece)} & \text{propagation through } d & \text{thin lens (objective)}
\end{bmatrix}
\]

\[
= \begin{pmatrix}
1 & -\frac{1}{fe} & 1 & -\frac{1}{f0}
\end{pmatrix}
\begin{pmatrix}
1 & 0
\end{pmatrix}
\begin{pmatrix}
1 & -\frac{1}{f0} & 1 & -\frac{1}{f0}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
1 - \frac{d}{fe} & \frac{d}{f0} - \frac{1}{fe} - \frac{1}{f0}
\end{pmatrix}
\begin{pmatrix}
\frac{f0}{fe} & 0
\end{pmatrix}
\]

\[
d = f0 + fe
\]

\[
\text{angular magnification}
\]

\[
\text{power} = 0
\]
Galilean Telescope

- Objective
- Eyepiece
- $f_0$
- $f_e$

Newtonian Telescope

- Image plane
- Paraboloidal primary mirror