LIGHT TRAPPING IN SOLAR CELLS

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Motivation: Solar Energy

- World energy demand is growing
- Fossil fuels are limited
- Solar energy is abundant and renewable
Need to be cheaper

- Technology exists, but is not yet implemented on a large scale
- Solar cells must compete on an open energy market
- Target = $1/W system price by 2017

Why Silicon Dominates

- **Silicon is:**
  - Abundant
  - Non-toxic
  - Easily processed
  - Ideal band gap
  - Pre-established infrastructure
High Efficiency Silicon Solar Cells

- 25% lab efficiency
- 400 µm thickness (150 – 250 µm typical)
- Wafer alone is 30 – 50% of cost!
- 100 cm² area per cell

Thin Film Silicon Solar Cells

- Reduce active layer thickness to 1 - 2 µm for lower material cost.
- Can be printed on large areas almost like paper (m² per cell)
- **PROBLEM**: Thin films are not very efficient

Uni-Solar PowerBond™, a-Si, 68 W $3.45/\text{W}$, ~6% efficient

![Electric Field Decay in Silicon](image)
Many tens to hundreds of microns in crystalline silicon. Depth grows as wavelength approaches bandgap.
- Roughly 1.0 kW / m²
- Silicon band gap around $\lambda_g = 1120$ nm
Consider a 1.0 µm slab of crystalline silicon above a PEC. Fraction of incident photons absorbed = 27%.
Anti-Reflective Coating (ARC)

- Perfect ARC improves things significantly (40%)
- Can we do better?

![Graph showing absorbance vs. wavelength for different coatings.](image-url)
Lambertian (Ergodic) Limit*

- Common benchmark and physical upper limit (for thick cells) (81%).
- How can we approach this limit?

Light Trapping in Thin Silicon Films

- Huge variety of ideas in literature
  - Surface Texturing
  - Distributed Bragg Reflectors
  - Surface Plasmonics


Nat. Mat, 9, 205 (2010)
APL, 89, 111111 (2006)
Model-N Slab

\[ A = 1 - R - T \]

Set \((R = 0)\)  
“N” for No reflections

\[ A = 1 - T \]
\[ T = 1 - A \]
\[ T = e^{-\alpha w} \]
Equivalent Path Length

Solve for thickness.

\[ \ell = \frac{1}{\alpha} \ln(1 - A) \]

Any arbitrary system with some absorption factor and attenuation coefficient can be expresses as equivalent model-N slab.
Slab-PEC Limits

\[ \ell_{\text{max}} = 2nw \quad \ell_{\text{min}} = \frac{2w}{n} \]

\[ \ell_N = 2w \]
Lambertian/Yablonovitch Limit
1. A ray (i.e., a plane wave) strikes the surface of the solar cell and scatters into the film.
2. Assume the scattering profile is perfectly diffuse (“Lambertian”).

\[ D(\theta, \phi) = 4 \cos(\theta) \]
3. Follow the rays as they propagate from top to bottom. Calculate how much light is absorbed and add up all the rays.

\[
A = \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi (1 - e^{-\alpha w \sec \theta})(4 \cos \theta) \sin \theta \, d\theta \, d\phi
\]
Lambertian/Yablonovitch Limit

\[ A = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} (1 - e^{-\alpha w \sec \theta})(4 \cos \theta) \sin \theta \, d\theta \, d\phi \]

3. Assume that the film is very thin and low-loss.

\[ \alpha w \ll 1 \]

\[ e^{-\alpha w \sec \theta} \approx 1 - \alpha w \sec(\theta) \]

\[ A \approx 2\alpha w \]

This one’s kinda sloppy. Justify me!
4. Calculate maximum possible light extraction efficiency.

$$\text{LEE} = \sin^2 \theta_c$$

$$\theta_c = \sin^{-1} \left( \frac{1}{n} \right)$$

$$\text{LEE} = \frac{1}{n^2}$$

- Lambertian incidence, perfectly random surface, perfect ARC.
- Snell’s Law
- Solve
5. Calculate total reflected power.

\[ R = (1 - 2\alpha w)(1 - 1/n^2) \]

\[ R \approx (1 - 1/n^2) \]

This thing is approximately 1. Ignore it.
6. Add up total absorbed power.

\[ A = \sum_{k=0}^{\infty} 2\alpha w \left( 1 - \frac{1}{n^2} \right)^k \]

\[ A = 2\alpha n^2 w \]
6. Add a mirror at the back. Convert to an equivalent path length.

\[ A = 4\alpha n^2 w \]

\[ \ell = 4n^2 w \]

\( n = 3.5 \) for crystalline silicon. That amounts to nearly 50 times the path-length enhancement!
Lambertian/Yablonovitch Limit


\[ \ell = 4n^2 w \]

n = 3.5 for crystalline silicon. That amounts to nearly 50 times the path-length enhancement!
Solar Concentration

- Near-perfect light trapping.
- Mechanical, physical, and economic limitations.
- What happens if lens is not aligned with incident rays?
Wave Guidance in Lossy Thin Films

- Light trapping is fundamentally a problem in guided mode coupling
- Need to better understand lossy waveguides

J.R. Nagel, S. Blair, and M. A. Scarpulla, “Exact field solution to guided wave propagation in lossy thin films” *Optics Express*, 19 (21), September 2011
Theory of wave guidance is rich and thorough... unless you add loss

Start with the vector Helmholtz equation (TE polarization)

\[ \nabla^2 E_y + k^2 E_y = 0 \]

Dispersion Relation:

\[ k_x^2 + k_z^2 = k_f^2 \]

Assume functional forms of electric/magnetic field intensities

\[
\begin{align*}
E(x, z) &= \hat{y} E_0 e^{+jk_z z} \\
&= \begin{cases} 
C e^{+j\gamma_c(x-h)} & (x > h) \\
C e^{+j\gamma_c(x-h)} + B e^{-jk_x x} & (|x| \leq h) \\
D e^{-j\gamma_s(x+h)} & (x < h)
\end{cases}
\end{align*}
\]

\[
\begin{align*}
H(x, z) &= \frac{E_0}{\omega \mu_0} e^{+jk_z z} \\
&= \begin{cases} 
C (\gamma_c \hat{z} - k_z \hat{x}) e^{+j\gamma_c(x-h)} & (x > h) \\
(\hat{z} k_x - \hat{x} k_z) e^{+jk_x x} - ((\hat{z} k_x + \hat{x} k_z) B e^{-jk_x x}) & (|x| \leq h) \\
-D (\gamma_s \hat{z} - k_z \hat{x}) e^{-j\gamma_s(x+h)} & (x < h)
\end{cases}
\end{align*}
\]
Enforce continuity at planar boundaries

Eigenvalue equation arises (much simpler for symmetric case):

\[ j \tan(2k_x h) = \frac{k_x (\gamma_c + \gamma_s)}{k_x^2 + \gamma_c \gamma_s} \]

\[ \gamma_c = j \sqrt{k_f^2 - k_c^2 - k_x^2} \]
\[ \gamma_s = j \sqrt{k_f^2 - k_s^2 - k_x^2} \]

Same as lossless case, but with complex values for \( k_x \)

Transcendental equations cannot be solved analytically

Apply steepest descent method to generate a solution

Residual:

\[ f_{TE}(k_x) = \tan(2k_x h) (k_x^2 + \gamma_c \gamma_s) + jk_x (\gamma_c + \gamma_s) \]

Misfit:

\[ \phi(k_x) = f(k_x)f^*(k_x) = ||f(k_x)||^2 \]
Example: Symmetric Lossy Waveguide (M = 2)

- \( n_f = 2.0 + j0.5 \)
- \( n_s = n_c = 1.5 \)
- \( h / \lambda_0 = 0.5 \)

Propagation Constants:

- \( k_x \lambda_0 = 7.89 + j0.77 \)
- \( k_z \lambda_0 = 9.89 + j3.38 \)
- \( \gamma \lambda_0 = -5.89 + j5.67 \)

*Nagel, Blair, & Scarpulla, Optics Express, 19 (21), September 2011*
Mode Profiles

- Mode profiles behave generally the same
- Extra mode solutions appear ($M = 3$)
- High loss = greater mode confinement

*Nagel, Blair, & Scarpulla, Optics Express, 19 (21), September 2011*
Cladding Loss

- Example: Add loss to cladding layer
  - \( n_f = 2.0 + j0.2 \)
  - \( n_s = n_c = 1.5 + j0.5 \)
  - \( \frac{h}{\lambda_0} = 0.5 \)

- Propagation Constants:
  - \( k_x \lambda_0 = 7.52 - j0.35 \)
  - \( k_z \lambda_0 = 10.15 + j1.82 \)
  - \( \gamma \lambda_0 = 2.2 + j5.07 \)
Wave Guidance in Amorphous Silicon

- Thin film amorphous silicon solar cell:
  - $n_f = 4.6 + j0.3$
  - $n_c = 1.0$, $n_s = 1.26 + j7.2$
  - $h = 500 \text{ nm}$, $\lambda_o = 600 \text{ nm}$

Nagel, Blair, & Scarpulla, *Optics Express*, 19(21), September 2011
Goos Hanchen Effect

- Field profile skews in the cladding layers
- Cladding propagation coefficient is complex, film is lossy.
- Remember Goos Hanchen effect

*Nagel, Blair, & Scarpulla, Optics Express, 19 (21), September 2011*
Branch Cuts

\[ \gamma = j \sqrt{k_f^2 - k_c^2 - k_x^2} \]

- Square-root in the dispersion relation has two solutions
- Multiple mappings to the misfit (Riemann Surface)
- Choose only convergent solutions

Positive Root

Negative Root

*Nagel, Blair, & Scarpulla, Optics Express, 19 (21), September 2011*
Loss Guidance (a-Si)

- Film loss introduces more eigenmode solutions
- Very lossy modes!

Nagel, Blair, & Scarpulla, Optics Express, 19 (21), September 2011
Anti-Guidance

- Use a real film index LOWER than the cladding:
  - \( n_f = 2.0 + j0.1 \)
  - \( n_s = n_c = 2.25 \)
  - \( h / \lambda_0 = 0.5 \)

- **Wave guidance still occurs!**