Optics for Energy
Week 12
Thursday

Statistical Ray Optics
This is a typical solar cell.

Brainstorm -> how can you improve it?
Geometrical optics for complex (random) surface shapes

3rd Assignment
November 25 in class
The goal is to make a technical case for your innovation. You need to address the following:
- Show simulations or calculations
- Show measurements/videos from prototype (or a mock-up with detailed drawings)
- Perform a simple cost-benefit analysis
- In each case above, list your assumptions clearly.
Motivation

Let's look at the highest efficiency solar module available today.
Let's look at the highest efficiency solar module available today.
MAXEON™ CELL TECHNOLOGY

NO WIRES TO BLOCK SUNLIGHT
Converts more energy

BACKSIDE MIRROR
Reflects more light

THICK PLATED COPPER CONTACTS
Conducts more current
Superficie texturizada
Silicio tipo N
**Basics**

**Assumption:** Behavior of light rays is ergodic. This means that once the ray enters a medium it forgets where and in which direction it came from. Statistically randomness.

**Results:**
- Only true for optically thick films $\lambda/2n$
- So far, we have ignored absorption.

In this situation, light gets an additional spot $I_0 = I_0 e^{-\alpha z}$.
Statistical ray optics

Eli Yablonovitch

Exxon Research & Engineering Company, P.O. Box 45, Linden, New Jersey 07036

Received December 17, 1981

A statistical approach is taken toward the ray optics of optical media with complicated nonspherical and nonplanar surface shapes. As a general rule, the light in such a medium will tend to be randomized in direction and of $2n^2(x)$ times greater intensity than the externally incident light, where $n(x)$ is the local index of refraction. A specific method for doing optical calculations in statistical ray optics will be outlined. These optical enhancement effects can result in a new type of antireflection coating. In addition, these effects can improve the efficiency as well as reduce the cost of solar cells.
Consider a textured optical sheet bathed in blackbody radiation.

Under thermal equilibrium, the electromagnetic energy density is given by Planck formula:

\[ U = \frac{\hbar \omega}{\exp \left( \frac{\hbar \omega}{kT} \right) - 1} \frac{2d\Omega \ k^2 \ dk}{(2\pi)^3} \]

Note that

\[ k = n \omega / c \]

Intensity is given by:

\[ I \equiv U \nu_g = \frac{\hbar \omega}{\exp \left( \frac{\hbar \omega}{kT} \right) - 1} \frac{2d\Omega \ n^2 \ \omega^2}{(2\pi)^3 \ c^2} \ d\omega. \]
The intensity within a medium in thermal equilibrium with a blackbody radiation is $n^2$ times higher than its environment.

$$I_{\text{int}}(\omega, x) = n^2(\omega, x) I_{\text{ext}}^{bb}(\omega).$$
Assumption: Behavior of light rays is ergodic. This means that once the ray enters a medium it forgets where and in which direction it came from. Statistical randomness.
We can show that this is true as long as the rays are ergodic, i.e., they forget their initial direction after one or two reflections (statistically averaged over the area of interest).
In this situation, light gets an additional pass.

\[ I_{\text{int}}(\omega, x) = 2n^2(\omega, x) I_{\text{inc}}(\omega) \]
Caveats:
Only true for optically thick films $\lambda/2n$
So far, we have ignored absorption.

So, for a wavelength of 1 micron and $n = 3.5$ (silicon), what is the smallest thickness this theory might be valid?

How much enhancement in intensity is possible?
Note on ergodic behavior. Consider a billiard table.

The number of angles accessible in a rectangular table (high symmetry) is only 2.
But with a polygonal table, ergodic behavior quickly appears.

But light is not like billiard balls & our systems are not closed. Light can escape if angle is smaller than critical angle.
\[ \sin \theta_c = \frac{1}{n} \]

The solid angle subtended by the escape cone is small.

\[ \Omega_c \approx \frac{1}{2n^2} \times 4\pi \]

The probability that a light ray inside the medium will escape is small. Or a small amount of randomness is required to create ergodic behavior.
Let $I_{inc}$ be the intensity of light incident on area element $dA$.

A fraction of the light, $T_{inc} (\phi)$, will be transmitted. The transmitted light must be balanced by the light that escaped = thermal equilibrium. This principle is also called detailed balance.
The internal intensity is isotropic and given by:

\[ I_{\text{int}} = \int B_{\text{int}} \cos \theta d\omega, \]

\[ I_{\text{int}} = 2 \times 2\pi \int_0^{\pi/2} B_{\text{int}} \cos \theta \sin \theta \, d\theta, \]

The factor 2 comes from the extra pass due to a reflective back-surface.

\[ I_{\text{int}} = 2\pi B_{\text{int}}. \]

The intensity of light that escapes is

\[ I_{\text{esc}} = 2\pi \int_0^\theta \frac{I_{\text{int}}}{2\pi} T_{\text{esc}}(\theta) \cos \theta \sin \theta \, d\theta, \]

\[ I_{\text{esc}} = I_{\text{int}} \frac{T_{\text{esc}}}{2n^2} \]
Now we can apply detailed balance. Light in = Light out.

\[ T_{\text{inc}}(\phi) \ I_{\text{inc}} = I_{\text{int}} \times \frac{T_{\text{esc}}}{2n^2} \]

\[ I_{\text{int}} = 2n^2 \times \frac{T_{\text{inc}}(\phi)}{T_{\text{esc}}} \times I_{\text{inc}}. \]

this factor suggests that one can exceed 2\(n^2\) (Note this factor is 1 for blackbody radiation)

\[ I_{\text{int}} = 2n^2 \times I_{\text{inc}}. \]

Possible but only if we restrict the incident angle. The brightness theorem still applies.
Light Trapping

Random textures are common in today's solar cells.

Why light trapping can overcome the limitations of certain materials, such as having higher quality optical simulations, lower reflection, and less carrier recombination.

Consider a light-trapping surface with varying textures and optical properties. The incident light interacts with the surface, causing absorption and reflection. The efficiency of the solar cell depends on how well the light is trapped and harvested.

Mathematical relations indicate that:

\[ \text{Transmitted intensity} = I_t = I_0 e^{-\alpha L} \]

Within the bandgap of Si, \( \alpha = 0 \),

\[ I_t (\text{absorbed}) = 0 \]

For different materials, the absorption coefficient \( \alpha \) varies, affecting the efficiency of light trapping.

\[ \text{Reflected intensity} = I_r = I_0 (1 - e^{-\alpha L}) \]

Optimizing the surface texture and material properties can significantly enhance the absorption and decrease reflection, improving the overall efficiency of solar cells.
Random textures are common in today’s solar cells
Why light trapping - can use thinner cells (cheaper, flexible, etc.) & can have higher quality semiconductor (fewer defects & less carrier recombination).
What happens when there is absorption?

The input light is \( A_{\text{inc}} I_{\text{inc}} T_{\text{inc}} \)

Light will escape at a rate \( \frac{A_{\text{esc}} I_{\text{int}} T_{\text{esc}}}{2n^2} \)

Light absorbed at interface (during reflection)

\[
\int_0^{\pi/2} \eta A_{\text{refl}} I_{\text{int}} \cos \theta \sin \theta \, d\theta = \frac{\eta A_{\text{refl}} I_{\text{int}}}{2}
\]

where \( \eta \) is the fractional absorption

Light is absorbed in the bulk

\[
\int \alpha \frac{I_{\text{int}}}{2\pi} \, dV \, d\Omega = \alpha l I_{\text{int}} A_{\text{inc}} \int_0^{\pi} \sin \theta \, d\theta = 2\alpha l I_{\text{int}} A_{\text{inc}}
\]
Light in = light out

\[ A_{\text{inc}} T_{\text{inc}} I_{\text{inc}} = \left( \frac{A_{\text{esc}} T_{\text{esc}}}{2n^2} + \frac{\eta A_{\text{refl}}}{2} + 2\alpha l \frac{A_{\text{inc}}}{A_{\text{inc}}} \right) I_{\text{int}} \]

\[ I_{\text{inc}} = \frac{T_{\text{inc}} I_{\text{inc}}}{\left( \frac{A_{\text{esc}} T_{\text{esc}}}{A_{\text{inc}} 2n^2} + \frac{\eta A_{\text{refl}}}{2A_{\text{inc}}} + 2\alpha l \right)} \]

Recall, the goal is to increase absorption in solar cell. This is given by:

\[ 2\alpha l \frac{A_{\text{inc}}}{A_{\text{inc}}} I_{\text{int}} = \frac{2\alpha l \frac{A_{\text{inc}}}{A_{\text{inc}}} T_{\text{inc}} I_{\text{inc}}}{\left( \frac{A_{\text{esc}} T_{\text{esc}}}{A_{\text{inc}} 2n^2} + \frac{\eta A_{\text{refl}}}{2A_{\text{inc}}} + 2\alpha l \right)} \]

Fraction on incoming light that is absorbed in the bulk

\[ f_{\text{vol}} \equiv \frac{2\alpha l \frac{A_{\text{inc}}}{A_{\text{inc}}} I_{\text{int}}}{A_{\text{inc}} I_{\text{inc}}} = \frac{2\alpha l T_{\text{inc}}}{\left( \frac{A_{\text{esc}} T_{\text{esc}}}{A_{\text{inc}} 2n^2} + \frac{\eta A_{\text{refl}}}{2A_{\text{inc}}} + 2\alpha l \right)} \]
Consider the case of weak absorption
Si is an indirect bandgap semiconductor.

$\Rightarrow \alpha$ is small.

Also assume $\gamma$ is small

This means fractional absorption due to imperfect reflections $\Rightarrow$ ideal case.
Also $T_{re} = T_{inc}$

Maximum absorption enhancement $= \frac{2 \times 2n^2}{4n^2}$ due to angle averaging
Polished side faces light. Rear surface is either flat or textured. Reflected light is measured by the integrating sphere.
 absorption band-edge red-shifted
Granular silicon sheets

Si microspheres are embedded in PMMA. Light is trapped in Si (due to higher index), enhancing absorption.
Detailed balance calculation in PMMA:
light from air→PMMA + Si→PMMA = PMMA→air + PMMA→Si

\[ \frac{A_{23} \bar{T}_{23} I_2}{2(n_2/n_3)^2} + A_{13} T_{13} I_1 = \left( \frac{A_{13} \bar{T}_{13}}{2n_3^2} + \frac{A_{23} \bar{T}_{23}}{2} \right) I_3, \quad 1=\text{air}, \ 2=\text{Si}, \ 3=\text{PMMA} \]

Detailed balance calculation in Si:
light from air→Si + PMMA→Si = Si→air + Si→PMMA + absorption in Si

\[ \frac{A_{23} \bar{T}_{23} I_3}{2} + A_{12} T_{12} I_1 \]

\[ = \left( \frac{A_{12} \bar{T}_{12}}{2n_2^2} + \frac{A_{23} \bar{T}_{23}}{2(n_2/n_3)^3} + 2\alpha l A_{12} \right) I_2. \]
$n_2 = 3.53, \ n_3 = 1.5$

$T_{12} \sim 0.96$ (with ARC)

$I_2 = \left( \frac{0.8316}{\alpha l + 0.0319} \right) \times 0.96 I_1,$

transmitted incident intensity

Within the bandgap of Si, $\alpha = 0$

$I_2 = \frac{0.8316 \times 0.96}{0.0319} I_1 = 2n_2^2 I_1$
Backscattered Light vs. Wavelength

- **No silicon granules**
- **With silicon granules**
3rd Assignment

November 25 in class

The goal is to make a technical case for your innovation. You need to address the following:

- Show simulations or calculations
- Show measurements/videos from prototype (or a mock-up with detailed drawings)
- Perform a simple cost-benefit analysis
- In each case above, list your assumptions clearly.