Basic properties of light

Light exists as a wave -> wavelength & frequency.
Light also is a particle -> momentum & energy.

\[ E = h\nu = hc/\lambda \]

Characteristics of light that are relevant to energy

- Spectral content of incident light
- Radiant power density of sunlight
- Angle at which the incident sunlight strikes absorber
- Radiant energy from sun throughout year
Basic properties of light

Energy of a photon

\[ E = h\nu = hc/\lambda \]

\( h = \) Planck’s constant = \( 6.626 \times 10^{-34} \) Js

We also express energy in electron-volts (eV). 1eV is the energy required to raise an electron through 1Volt potential.

\( 1 \text{ eV} = 1.602 \times 10^{-19} \) J

What is the energy of a green photon (wavelength = 500nm) ?

What is the energy of an infra-red photon (wavelength = 1micrometer) ?

Photon Flux

Photon flux is defined as the number of photons per second per unit area. This determines the number of electrons generated in a solar cell, for instance.

\[ \Phi = \# \text{ of photons} / (\text{time} \times \text{area}) \quad [\text{s}^{-1}\text{m}^2] \]
Basic properties of light

But photon flux doesn't give any information on photon energy. So power density is calculated by multiplying photon flux by the energy of a single photon.

\[ H \ (W/m^2) = \Phi \times \frac{hc}{\lambda} \]  
Note that this is wavelength dependent.

If a red beam has the same power density as a blue beam, which beam has higher photon flux?

If both beams are incident on a solar cell, which will produce higher current? Which will produce more electric power?
The concept of a "ray"

wavefronts
(planes of constant phase)
The concept of a "ray"

In homogeneous media, light propagates in rectilinear paths.

Inhomogeneous media: light propagates in helical paths.

$t = 0$

$t = \Delta t$

direction of energy flow = ray

$\lambda$
Reflection

- \( \theta_i = \theta_r \)
- Normal, incident & reflected rays lie in one plane
Refraction

- Incident, reflected & refracted rays lie in one plane

\[ n_1 \sin \theta_i = n_2 \sin \theta_t \]

Snell's Law

- Incident, reflected & refracted rays lie in one plane
Total Internal Reflection

- \( n_1 > n_2 \)
- when \( \theta_i = \theta_c = \sin^{-1} \frac{n_2}{n_1} \)
  \( \theta_t = 90^\circ \)
- when \( \theta_i > \theta_c \)
  all light is reflected
Analyzing Lenses: Ray Tracing

- Point source (object)
- Free space propagation in air
- Refraction at air-glass interface
- Free space propagation in glass
- Refraction at glass-air interface
- Free space propagation in air
- Optical axis
- Point image

Diagram shows the path of a ray through a lens, from point source through air to glass, then through another air interface to form an image on the other side.
Paraxial Approximation

- Only rays close to the optical axis are considered
  \[ \varepsilon \ll 1 \text{ rad} \]
- 1st order Taylor approximations apply
  \[ \sin \varepsilon \approx \varepsilon \quad \tan \varepsilon \approx \varepsilon \quad \cos \varepsilon \approx 1 \]
- Valid for \( \varepsilon \) upto 10-30 degrees
Matrix Formulation

\[
\begin{pmatrix}
  n_{\text{out}} & \alpha_{\text{out}} \\
  x_{\text{out}}
\end{pmatrix} =
\begin{pmatrix}
  M_{11} & M_{12} \\
  M_{21} & M_{22}
\end{pmatrix}
\begin{pmatrix}
  n_{\text{in}} & \alpha_{\text{in}} \\
  x_{\text{in}}
\end{pmatrix}
\]

Translation through Uniform Medium

\[
\begin{pmatrix}
  n & \alpha_1 \\
  x_1
\end{pmatrix} =
\begin{pmatrix}
  1 & 0 \\
  \frac{D_{01}}{n} & 1
\end{pmatrix}
\begin{pmatrix}
  n & \alpha_0 \\
  x_0
\end{pmatrix}
\]

Refraction by Spherical Surface

\[
\begin{pmatrix}
  n' & \alpha'_1 \\
  x'_1
\end{pmatrix} =
\begin{pmatrix}
  1 & -\left(\frac{n' - n}{R}\right) \\
  0 & 1
\end{pmatrix}
\begin{pmatrix}
  n & \alpha_1 \\
  x_1
\end{pmatrix}
\]
Thin Lens

$n = 1$

$n'$

optical axis

$R = \text{radius of curvature}$

Refraction at 2nd spherical interface

Refraction at 1st spherical interface

$\alpha'_{\text{out}}$

$\alpha_{\text{in}}$

$\chi'_{\text{out}}$

$\chi_{\text{in}}$
Thin Lens

\[
\begin{pmatrix}
\alpha'_{\text{out}} \\
\chi'_{\text{out}}
\end{pmatrix} =
\begin{pmatrix}
1 & -\left(\frac{1 - n'}{R'}\right) \\
0 & 1
\end{pmatrix}
\times
\begin{pmatrix}
1 & -\left(\frac{n' - 1}{R}\right) \\
0 & 1
\end{pmatrix}
\times
\begin{pmatrix}
\alpha_{\text{in}} \\
\chi_{\text{in}}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
1 & -\left(\frac{n' - 1}{R} + \frac{1 - n'}{R'}\right) \\
0 & 1
\end{pmatrix}
\]

\[
P_{\text{thin-lens}} = (n' - 1)\left(\frac{1}{R} - \frac{1}{R'}\right)
\]

Lens-maker's formula
Power of Surfaces

- Matrix Formulation

\[ \text{Power} = - M_{12} \]

- Power & Focal Length

\[ f = \frac{1}{\text{power}} \]
Note: In paraxial approximation, principal and focal planes are flat, whereas in reality these are curved surfaces (not spherical).
Thick Lens: Focal Lengths

FFL = Front Focal Length
BFL = Back Focal Length
EFL = Effective Focal Length
Thick Lens: Matrix Transformation

\[
\begin{bmatrix}
\alpha'_{\text{out}} \\
x'_{\text{out}}
\end{bmatrix}
= 
\begin{bmatrix}
1 & -\left(\frac{1 - n'}{R_2}\right) \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
\frac{D_l}{n'} & 1
\end{bmatrix}
\begin{bmatrix}
1 & -\left(\frac{n' - 1}{R_1}\right) \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\alpha_{\text{in}} \\
x_{\text{in}}
\end{bmatrix}
\]
Thick Lens: Matrix Transformation

\[ \alpha'_{\text{out}} = 1 + \frac{D_l}{n'} \left( \frac{n' - 1}{R_2} \right) \]

\[ - (n' - 1) \left\{ \frac{1}{R_1} - \frac{1}{R_2} + (n' - 1) \frac{D_l}{n'R_1R_2} \right\} \]
Thick Lens: Matrix Transformation

optical axis

Dl

refraction (radius of curvature = R₁)

propagation through Dl

refraction (radius of curvature = R₂)

EFL = f

\[ \frac{1}{f} = (n' - 1)\left\{ \frac{1}{R₁} - \frac{1}{R₂} + (n' - 1)\frac{Dl}{n'R₁R₂} \right\} \]
Generalized Imaging Conditions

\[
\begin{pmatrix}
  n' \\ x'
\end{pmatrix}
= 
\begin{pmatrix}
  M_{11} & M_{12} \\
  M_{21} & M_{22}
\end{pmatrix}
\begin{pmatrix}
  n \\ x
\end{pmatrix}
\]

- **Image**
- **System Matrix**
- **Object**

Power \( P = -M_{12} \)

Imaging Condition \( M_{21} = 0 \)

Lateral Magnification \( m_x = M_{22} \)

Angular Magnification \( m_a = (n/n')M_{11} \)
Prisms
Assume a symmetric case,

\[ a = b \]
\[ a' = b' \]

From Snell's law,

\[ a' = \frac{\theta}{2} \]
\[ a = \frac{\theta + D}{2} \]

Prism Equation

\[ n = \frac{\sin\left(\frac{D + \theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)} \]
Dispersion

Refractive index $n$ is a function of the wavelength.

Newton's prism

- Red
- Green
- Blue

White light (all visible wavelengths)

Air

Glass

Newton's prism
Dispersion measures

Reference color lines
C (H- $\lambda$=656.3nm, red), D (Na- $\lambda$=589.2nm, yellow),
F (H- $\lambda$=486.1nm, blue)

Crown glass has

$$n_F = 1.52933 \quad n_D = 1.52300 \quad n_C = 1.52042$$

Dispersive power

$$V = \frac{n_F - n_C}{n_D - 1}$$

Dispersive index

$$v = \frac{1}{V} = \frac{n_D - 1}{n_F - n_C}$$
Example: spherical mirror

In the paraxial approximation, it (approximately) focuses an incoming parallel ray bundle (from infinity) to a point.
Reflective optics formulae

Imaging condition
\[ \frac{1}{D_{12}} + \frac{1}{D_{01}} = -\frac{2}{R} \]

Focal length
\[ f = -\frac{R}{2} \]

Magnification
\[ m_x = -\frac{D_{12}}{D_{01}} \quad m_\alpha = -\frac{D_{01}}{D_{12}} \]
Telescope: Matrix Formulation

Objective eyepiece

\[ f_0 \]

\[ f_e \]

\[ y' \]

\[ D \]

System matrix =

\[
\begin{align*}
\begin{bmatrix}
\text{thin lens} \\
(eyepiece)
\end{bmatrix} & \begin{bmatrix}
\text{propagation} \\
\text{through } d
\end{bmatrix} & \begin{bmatrix}
\text{thin lens} \\
(objective)
\end{bmatrix}
\end{align*}
\]

\[
= \begin{bmatrix}
1 & -\frac{1}{f_e} \\
0 & 1
\end{bmatrix} \begin{bmatrix}
1 & 0 \\
d & 1
\end{bmatrix} \begin{bmatrix}
1 & -\frac{1}{f_0} \\
0 & 1
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\frac{1-d}{f_e} & \frac{d}{f_0} - \frac{1}{f_e} - \frac{1}{f_0} \\
d & 1 - \frac{d}{f_0}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
-\frac{f_0}{f_e} & 0 \\
d & -\frac{f_e}{f_0}
\end{bmatrix}
\]

\[ d = f_0 + f_e \]

Angular magnification

Power = 0
Similarly, we can keep adding mirrors. And these mirrors can be infinitesimally small. This means that we can trace out a reflective surface (curve).

Note the symmetry about the vertical axis.
The angle, beta has to be minimized at each point on this curve.
This is also the curve that produces the smallest beta & largest entrance aperture C3D3.
All edge rays must get reflected onto A. The curve has to be a parabola. The parabola has its axis parallel to the edge rays and its focus at A = F.
If we keep extending the parabolae, they curve inward and entrance aperture is decreased. 
$C_4D_4 < CD$. 
Shadowing occurs.
Edge-ray principle = light rays coming from the edges of the source must be deflected onto the edges of the receiver. This is the basic design principle of non-imaging concentrators.

So all rays entering CPC with input angle $< \theta$ are trapped & end up at the receiver. But all rays entering CPC with input angle $> \theta$ are lost (reflected back).
Acceptance = \frac{\text{# of rays reaching receiver ("accepted")}}{\text{# of rays entering CPC}}

= \left\{ \begin{array}{l}
1 & -\alpha < \alpha_i < \alpha \\
0 & \alpha_i > \alpha
\end{array} \right.

\theta = \text{half-acceptance angle.
For a general parabola,

\[ AC + CF = BD + DF \]

For the CPC - BDE

\[ CB + a_2 = ED + DA \]
\[ CB = DA \]
\[ \Rightarrow a_2 = ED = a_1 \sin \theta \]
\[ \Rightarrow \frac{a_1}{a_2} = \frac{1}{\sin \theta} \]

The optical path length between the wavefront CE & focus A is same for all edge rays perpendicular to CE.
As \( \theta \uparrow, h \downarrow \)
\( \theta \to 0, \ h \to \infty \)

So, for small acceptance angles, the CPC becomes very tall.