Transient & AC conditions

So far, we have looked at pn junctions at steady state. But most such devices are used under AC or for switching. So we need to have some basic understanding of how things behave transiently.

Let's look at the switching of a diode from forward to reverse bias in detail.
The variation of stored charge

Under steady carrier injection, we have this charge distribution.

If the current changes, we can expect the charges to change as well. But it takes time for the charges to change. In other words, charges will lag the current. This is a capacitive effect of the pn junction.

We need to solve the time-dependent continuity equations for the current.

From chapter 4,

\[
\frac{\partial p(x,t)}{\partial t} + \frac{\partial}{\partial x} \left( q \frac{\partial p(x,t)}{\partial x} \right) = \frac{q}{\tau_n} \left( \delta n(x) - n(x) \right)
\]

The hole current as a function of time and space is

\[
\frac{\partial p(x,t)}{\partial t} = q \left( \frac{\delta p(x,t)}{\partial x} + \frac{\delta p(x,t)}{\partial t} \right)
\]

The current density at an instant of time is then given by

\[
J_t(x,t) = q \int_0^t \left( \frac{\delta p(x,t)}{\partial x} + \frac{\delta p(x,t)}{\partial t} \right) dx
\]

If we further assume that current injection is from p+ region to a long n region, then the total current is mostly hole current at \( x = 0 \).

Also, \( J_t(x,t) = 0 \).

Then, the total current as a function of time is

\[
J(x,t) = J_t(x,t) = q A \int_{-\infty}^t \delta p(x,t) dx + q A \int_0^t \delta p(x,t) dx + q A \int_0^t \delta p(x,t) dx_t
\]

\[
\frac{d}{dt} \int J(x) dx + \int [\delta p(x,t) + \delta p(x,t)] dx_t
\]

recombination term - excess carriers need to be replenished.

This equation can be solved numerically given the initial conditions. Let's look at an example.
We need to solve the time-dependent continuity equations for the current.

\[
\frac{\partial p(x, t)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p}
\]

From chapter 4,

\[
\frac{\partial \delta n}{\partial t} = \frac{1}{q} \frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}
\]

The hole current as a function of time and space is

\[
-\frac{\partial J_p(x, t)}{\partial x} = q \frac{\delta p(x, t)}{\tau_p} + q \frac{\partial p(x, t)}{\partial t}
\]

The current density at an instant of time is then given by

\[
J_p(0) - J_p(x) = q \int_0^x \left[ \frac{\delta p(x, t)}{\tau_p} + \frac{\partial p(x, t)}{\partial t} \right] dx
\]
If we further assume that current injection is from p+ region to a long n region, then the total current is mostly hole current at $x_n=0$.

Also, $J_p$ at $x_n=\infty$ is 0.

Then, the total current as a function of time is

$$i(t) = i_p(x_n = 0, t) = \frac{qA}{\tau_p} \int_0^\infty \delta p(x_n, t) dx_n + qA \frac{\partial}{\partial t} \int_0^\infty \delta p(x_n, t) dx_n$$

$$i(t) = \frac{Q_p(t)}{\tau_p} + \frac{dQ_p(t)}{dt}$$

charge buildup (or depletion) term. This term is 0 for steady-state.

recombination term = excess carriers need to be replenished.

This equation can be solved numerically given the initial conditions. Let’s look at an example.
It takes some time for the excess holes to recombine with the electrons and the total charge to go to 0.

Solving the equation using the Laplace transform:

\[ 0 = \frac{1}{\tau_p} Q_p(s) + sQ_p(s) - I\tau_p \]

\[ Q_p(s) = \frac{I\tau_p}{s + 1/\tau_p} \]

\[ Q_p(t) = I\tau_p e^{-t/\tau_p} \]

initial total charge

hole lifetime in n-type region
Even though the current is turned off, the charge and hence, a potential persists across the junction.

\[ \Delta p_n(t) = p_n(e^{q\nu(t)/kT} - 1) \]

But, we can apply the quasi-steady state approximation & assume that the excess carrier distribution is exponential.

\[ \delta p(x_n, t) = \Delta p_n(t)e^{-x_n/L_p} \]

The instantaneous stored charge is given by

\[ Q_p(t) = qA \int_0^\infty \Delta p_n(t)e^{-x_n/L_p}dx_n = qAL_p\Delta p_n(t) \]

\[ \Delta p_n(t) = p_n(e^{q\nu(t)/kT} - 1) = \frac{Q_p(t)}{qAL_p} \]

\[ \nu(t) = \frac{kT}{q} \ln \left( \frac{I\tau_p}{qAL_pP_n e^{-t/\tau_p} + 1} \right) \]

This simple model tells us that we cannot instantaneously switch voltage off in a pn junction diode due to the excess stored charge.

If we use a p+n junction with the n region smaller than the hole diffusion length, then very little charge is stored. = narrow base diode.

One can also add impurities to increase recombination, so that the charges are dissipated faster for faster switching.
Reverse recovery transient

What does the current transient look like when the diode is switched from forward to reverse and back?

What is the forward bias current here?

What is the instantaneous reverse bias current at t=0?

The charges stored here during forward bias (carrier injection) needs to be discharged during reverse bias. Junction voltage can't change instantaneously!
The stored charge (& junction voltage) can't change instantaneously. So this is \(-E/R\).
voltage across junction remains small (same as during forward bias)
minority carrier concentration near the junction

\[ \delta p(x_n, t) \]

\[ \Delta p_n(t) = p_n(e^{\frac{qv(t)}{kT}} - 1) \]

\[ t \ll T/2 \]

the slope must be >0

this gives us the junction voltage.

here, the stored charges are depleted

here, the junction voltage < 0.
\[ \Delta p_n(t) = p_n(e^{qv(t)/kT} - 1) \]

The stored charge is depleted here.

As more voltage drops across the junction, the current decreases until it reaches the reverse saturation current.
Reverse saturation current

Storage delay time = time to discharge all junction charge.

This limits how fast the diode can be switched!

What is the output of an ideal rectifier?

\[ t_{sd} = \tau_p \left[ \text{erf}^{-1} \left( \frac{I_f}{I_f + I_r} \right) \right]^2 \]
switching voltage

resulting current

this is the capacitive effect due to the time required to discharge stored charges in the junction. ---> limits switching speed.
Capacitance of pn junction

What is capacitance?
The ability to store charge.
See background video for parallel-plate capacitor.

For a pn junction, there are two types of capacitance.

**Junction capacitance**
doninant under reverse bias

- The capacitance is:
  \[ C = \frac{|dQ|}{dV} \]
- The depletion region width under bias is:
  \[ W = \left( \frac{3.3 \times 10^{-3} N_D + N_A}{Q} \right)^{1/3} \]
- The charge in the depletion region is:
  \[ q = -\frac{N_D N_A}{N_D + N_A} \]
- The capacitance is:
  \[ C = \frac{|dQ|}{dV} = \frac{\pi}{2} \left( \frac{N_D N_A}{N_D + N_A} \right)^{1/3} \]

**Charge storage capacitance**
doninant under forward bias

The charge storage capacitance arises from the delay between current & voltage (also called diffusion capacitance).

- In a large diode (most direct bandgap semiconductors), the carrier lifetime is short and the delay between current and voltage under forward bias is small — diffusion capacitance is negligible.
- If the diode is short compared to the diffusion lengths (most Si pn junction diodes), then we can estimate the diffusion capacitance (due to stored holes on n-side) as:
  \[ C_1 = \frac{Q_D}{dV} = \frac{1}{3} \frac{q^2}{kT A p V_0 W l_{diff}} \]

A related parameter is the ac conductance. For a long diode, we have:
\[ C_2 = \frac{dG}{dV} = -\frac{2 \pi e \omega}{\tau} \frac{dG}{dV} = \frac{G}{\omega^2} \]
Junction capacitance

The capacitance is

$$C = \left| \frac{dQ}{dV} \right|$$

The depletion region width under bias is

$$W = \left[ \frac{2\varepsilon(V_0 - V)}{q} \left( \frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2}$$

$$|Q| = qA x_{n0} N_d = qA x_{p0} N_a$$

$$x_{n0} = \frac{N_a}{N_a + N_d} W, \quad x_{p0} = \frac{N_d}{N_a + N_d} W$$

$$|Q| = qA \frac{N_d N_a}{N_d + N_a} W = A \left[ 2q\varepsilon(V_0 - V) \frac{N_d N_a}{N_d + N_a} \right]^{1/2}$$

$$C_j = \left| \frac{dQ}{d(V_0 - V)} \right| = A \left[ \frac{2q\varepsilon}{(V_0 - V)} \frac{N_d N_a}{N_d + N_a} \right]^{1/2}$$
Voltage-dependent capacitance.

\[ C_j = \varepsilon A \left[ \frac{q}{2\varepsilon(V_0 - V)} \left( \frac{N_d N_a}{N_d + N_a} \right) \right]^{1/2} = \frac{\varepsilon A}{W} \]

When one side much more heavily doped,

\[ C_j = \frac{A}{2} \left( \frac{2q\varepsilon}{V_0 - V} \frac{N_d}{N_d} \right)^{1/2} \quad \text{for } p^+-n \]

If we measure this, we can estimate this.

Behavior under reverse bias.
The charge storage capacitance arises from the delay between current & voltage (also called diffusion capacitance).

In a long diode (most direct bandgap semiconductors), the carrier lifetime is short and the delay between current and voltage under forward bias is small --> diffusion capacitance is negligible.

If the diode is short compared to the diffusion lengths (most Si pn junction diodes), then we can estimate the diffusion capacitance (due to stored holes on n-side) as

$$C_s = \frac{dQ_p}{dV} = \frac{1}{3} \frac{q^2}{kT} Acp_{n} \epsilon e^{qV/kT}$$

The diffusion capacitance depends upon how the excess carriers are removed from the device, hence numerical modeling is required for accurate values.

A related parameter is the ac conductance. For a long diode, we have

$$G_s = \frac{dI}{dV} = \frac{qAL_p p_n}{\tau_p} \frac{d}{dV} \left( e^{qV/kT} \right) = \frac{q}{kT} I$$
Capacitance of pn junction

What is capacitance?
The ability to store charge.
See background video for parallel-plate capacitor.

For a pn junction, there are two types of capacitance.

Junction capacitance

- dominant under reverse bias

\[ C_J = \frac{dQ}{dV} \]
\[ W = \left( \frac{2W_1V_1}{N_1N_2} \right) \]
\[ \text{The capacitance is} \]
\[ \text{The depletion region width under bias is} \]
\[ \text{the voltage dependence of the capacitance} \]
\[ C_J = \left[ \frac{dQ}{dV} - V \right] = \left[ \frac{1}{2(1 + V/3)N_1N_2} \right] \]

Charge storage capacitance

- dominant under forward bias

\[ C_C = \frac{dQ}{dV} = \frac{1}{3} \frac{\tau}{kT} \]
\[ \text{The charge storage capacitance arises from the delay between current & voltage (also called diffusion capacitance).} \]

In a long diode (most direct bandgap semiconductors), the carrier lifetime is short and the delay between current and voltage under forward bias is small — diffusion capacitance is negligible.

If the diode is short compared to the diffusion lengths (most npn junction diodes), then we can estimate the diffusion capacitance (due to stored holes on n-side) as

\[ C_C = \frac{dQ}{dV} = \frac{\tau}{kT} \]

A related parameter is the ac conductance. For a long diode, we have

\[ G_C = \frac{dI}{dV} = \frac{\tau}{kT} \frac{dV}{dV} \]
Thus, the pn junction diode forms a voltage-controlled capacitor.

This is extremely useful for tuning circuits.